

DOI: [10.52950/ES.2023.12.2.005](https://doi.org/10.52950/ES.2023.12.2.005)

AN ERROR IN THE THEORY OF SEIGNIORAGE

PETR MACH

Abstract:

The article reveals an error in the theory of seigniorage or government revenue from printing money. Friedman, Cagan, Mankiw, and other economists have repeated the definition of seigniorage as the monetary base multiplied by the rate of inflation (or by the growth in the monetary base). This definition, however, contains a logical error in the formula of seigniorage, confusing the growth rate with the growth rate divided by the growth rate plus one. This often-repeated confusion is consequently common in modern economics textbooks. These many authors have omitted Keynes who – almost one hundred years ago – rightly understood seigniorage to be the monetary base multiplied by the rate of money growth divided by the rate of money growth plus one. This paper presents an unambiguous definition and formula of seigniorage and offers a graphical illustration of seigniorage as a function of monetary growth which corresponds to Keynes's approach. It is shown that such a graph is an analogy to the Laffer curve in explicit taxation.

Keywords:

Monetary Theory; Monetary Base; Seigniorage; Inflation Tax; Laffer curve

JEL Classification: E00, E69, H00

Authors:

PETR MACH, University of Finance and Administration, Czech Republic, Email: petrmach1975@gmail.com

Citation:

PETR MACH (2023). An Error in the Theory of Seigniorage. International Journal of Economic Sciences, Vol. XII(2), pp. 83-91., [10.52950/ES.2023.12.2.005](https://doi.org/10.52950/ES.2023.12.2.005)

1. Introduction

The idea that printing money or expansionary monetary policy increases prices is well known and is supported not only by theory (the quantity theory of money) but also by empirical studies. For instance, Čermáková and Filho (2021) showed recently the effect of expansionary monetary policy on the prices of agricultural commodities.

Increased prices due to monetary expansion come at the expense of consumers and usually in favour of the government which benefits from an increase in the monetary base through various ways. Printing money transfers wealth from the public to the government like an explicit tax.

In post-covid years many economies throughout the world once again experience relatively high rates of inflation which makes the topic of “inflation tax” up-to-date again.

The theory of seigniorage or inflation tax can be briefly summarized as follows: By printing money, the monetary authority, typically the central bank inflates the price level, which means that it decreases the purchasing power of money. The consequence of printing money is a stealthy transfer of wealth from the public to the government. It is like imposing a tax on all money holders. This “inflation tax” is also known as seigniorage.

There are several concepts of seigniorage in economic literature. For instance, Klein and Neumann (1990) distinguish three basic concepts of seigniorage, namely the monetary seigniorage, the opportunity-cost seigniorage, and the fiscal seigniorage each with a slightly different definition.

In this paper, we will deal specifically with what is usually referred to as the monetary seigniorage. This concept links seigniorage to the growth rate of the monetary base or to the rate of inflation, which are of the same value in a “steady state.”

In this article, the author reveals an error rooted in economic literature for the past seventy years. It will be demonstrated that many respected economists have been using a wrong formula of seigniorage and have omitted Keynes’s early work on inflation tax.

2. Monetary seigniorage in economics literature

Most economic articles on the monetary concept of seigniorage assume that seigniorage is calculated as the product of money balances and the rate of inflation. Alternatively, instead of by the rate of inflation, the monetary base is multiplied by the rate of growth of the monetary base. The rate of inflation is equal to the growth rate of the monetary base in a steady state (a situation where there is no economic growth, no change in the velocity of circulation of money, and no change in the money multiplier), as it follows from the quantity theory of money.¹

¹ The “equation of exchange” of the quantity theory of money says that the money stock (M) times the income velocity of circulation of money (V) equals the average price (P) times output (Y): $M \cdot V = P \cdot Y$. The money stock is

It will be demonstrated that this concept of seigniorage assuming that seigniorage is the product of money balances times the rate of inflation or the rate of the growth of the monetary base is logically wrong, and that with few exemptions (namely Easterly et al. 1995 and Walsh 2010) most authors repeat the wrong definition coined by Chicago economists Milton Friedman and Phillip Cagan in 1950s.

The author agrees that it is completely correct to analyse seigniorage as a tax. As the revenue of the income tax is the product of a tax base (income) and a rate, and as we can also calculate income tax as a percentage of GDP, we can do the same for seigniorage. The author of this article, however, claims that many economists were wrong regarding what should be the rate of the seigniorage tax and that the rate of inflation is not the correct value of the rate in this tax.

Consider articles on seigniorage written mostly by economists from the University of Chicago in the 1950s.

In 1953 Milton Friedman published the article "Discussion of the Inflationary Gap". He wrote regarding seigniorage: "*The price rise imposes a tax on the holding of [monetary units]. The proceeds of this tax can be garnered by the government...A price rise of 10 per cent per year is precisely equivalent...to a stable price level plus a tax of 10 per cent per year on the average amount of cash balances.*" (Friedman 1953, p. 254-5)

In 1956 another Chicago monetarist, Phillip Cagan, followed with his article "Monetary Dynamics of Hyperinflation", the most cited article on seigniorage and inflation tax since then. "*Issuing money was a method of raising revenue by a special kind of tax – a tax on cash balances...The base of the tax is the level of real cash balances; the rate of the tax is the rate of depreciation in the real value of money, which is equal to the rate of rise in prices*" (Cagan, 1956, p. 78). He provided a formula for such a tax as the product of the base (real money balances) and the rate (rate of inflation)

$$\frac{M}{P} \left(\frac{dP}{dt} \frac{1}{P} \right) \quad (1)$$

Other Chicago economists followed Cagan's approach. Martin Bailey writes "*The revenues...derived by the government at any constant rate of inflation...,to which the public has fully accommodated itself, will be equal to the rate of inflation times the real cash balances then held by the public.*" (Bailey 1956, p. 102)

Later Milton Friedman wrote on seigniorage, "*The rate of price rise is the rate of the tax. The real stock of money is the base of the tax. By strict analogy with an excise tax on a commodity, the yield is the product of the two.*" (Friedman 1972, p. 846-7)

the monetary base (MB) times the money multiplier (m). It extends the equation of exchange into $MB \cdot m \cdot V = P \cdot Y$. In an equation, any percentage change on the left side must be equal to the percentage change on the right side. Under a steady state, m, V, and Y in the equation of exchange are constant, and therefore, any percentage change in the monetary base is exactly equal to the percentage change in the price level.

The assumption that seigniorage is the product of the monetary base and the rate of inflation or money growth was consequently repeated in many economics textbooks around the world.

Peter Bofinger, a German economist, in his book *Monetary Policy*, writes “*seigniorage is obtained as the product of growth rate in the monetary base and the real monetary base, $\frac{B_t - B_{t-1}}{B_{t-1}} \cdot \frac{B_{t-1}}{P_{t-1}}$.” (Bofinger 2001, p. 371)*

Similarly, Olivier Blanchard, a French economist, writes in his *Macroeconomics*: “*Seigniorage is the product of money growth times real money balances*” (Blanchard 1997, p. 429).

Gregory Mankiw of Harvard University says “*the real revenue raised from seigniorage is $\frac{\Delta M}{P} = \frac{\Delta M}{M} \cdot \frac{M}{P}$ ” (Mankiw 1987, p. 329) and he offers an illustrative example of this concept in his book *Macroeconomics*: “*a drug dealer holding \$20,000 in cash pays an inflation tax of \$2,000 per year when inflation rate is 10 percent.*” (Mankiw 2003, p. 492).*

2.1 Cagan’s mistake

This whole concept of seigniorage is, however, based on simplification or maybe a mistake. Cagan’s statement that “*depreciation in the real value of money...is equal to the rate of rise in prices*” is not correct and nor is Friedman’s “*A price rise of 10 per cent per year is precisely equivalent...to ...a tax of 10 per cent per year on the average amount of cash balances.*” [emphases added, both quoted above]

Let us explain this mistake in an example. Assume we have a one-hundred-dollar bill. Assume that a Big Mac costs four dollars. We can buy 25 Big Macs with our bill. Now assume there is an inflation of 25 per cent so that the price of one Big Mac goes from four to five dollars. Our one-hundred-dollar bill will now buy only 20 Big Macs, which is 20 per cent less than 25 Big Macs before. In other words, a 25 per cent inflation is equivalent to a constant price level and a tax of 20 per cent on our money holdings. It is like a Big Mac still cost four dollars, and the government took twenty dollars from our one-hundred-dollar bill.

In general, the rate of depreciation of money or the percentage change in the purchasing power of money is always lower than the rate of inflation and is equal to

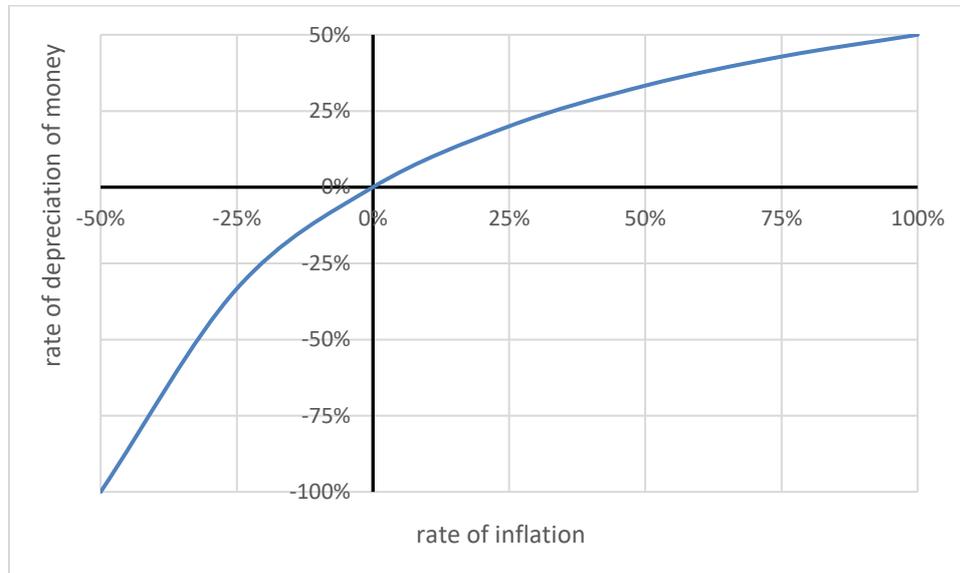
$$\delta = \frac{\pi}{1+\pi} \quad (2)$$

where δ is the percentage change in the value of money (purchasing power of money – or the rate of depreciation of money) and π is the percentage change in the price level (the rate of inflation).

Fig. 1 shows that the change in the purchasing power of money and the rate of inflation are not the same. For instance, with the rate of inflation of 100%, the purchasing power of money depreciates by 50%. With the rate of inflation of –50%, the purchasing power of money appreciates by 100% or the rate of depreciation of money is –100%. For

small rates of inflation, the rate of depreciation of money is approximately the same. For instance, inflation of 1% means depreciation of money of 0.9901%.

Fig. 1: The change in the purchasing power of money vs inflation



2.2 Keynes was right

In 1924 John M. Keynes in his almost-forgotten book “Tract on Monetary Reform” wrote:

“Let us suppose that there are in circulation 9,000,000 currency notes, and that they have altogether a value equivalent to 36,000,000 gold dollars. Suppose that the Government prints further 3,000,000 notes, so that the amount of currency is now 12,000,000; then in accordance with the...theory, the 12,000,000 notes are still only equivalent to \$36,000,000. In the first state of affairs, therefore each note = \$4, and in the second state of affairs each note = \$3. Consequently the 9,000,000 notes originally held by the public are now worth \$27,000,000 instead of \$36,000,000, and the 3,000 notes newly issued by the Government are worth \$9,000,000. Thus by the process of printing the additional notes the Government has transferred from the public to itself an amount of resources equal to \$9,000,000, just as successfully as if it had raised this sum by taxation. On whom has this tax fallen? Clearly on the holders of the original 9,000,000 notes, whose notes are now 25 per cent less than they were before. The inflation has amounted to a tax of 25 per cent on all holder of notes in proportion to their holdings.” (Keynes 1924, p. 42-43)

Keynes correctly found that increasing the amount of money by one third (from 9 to 12 million) yields a tax on money holdings of one quarter (a fall from 36 to 27 million gold dollars). If Mankiw followed Keynes’s logic instead of Cagan’s, his above-mentioned example of a drug dealer would give a different result. If the dealer holds 20,000 in cash and the government creates inflation of 10 per cent it is equivalent to a tax rate on money holdings of 9.09 per cent ($\frac{0.1}{1+0.1}$). It is as if the government took \$1818 and not \$2000, as Mankiw claims, out of his \$20,000 cash.

Friedman and Cagan either made a deliberate simplification or made an unintended logical mistake. What is apparent, is that in their works, Friedman, Cagan, Mankiw and others have not referred to Keynes's work of 1924. Had they been inspired by him, the theory of seigniorage might not have been based on a mistake today.

Not all economists, however, repeated Friedman's and Cagan's mistake. Carl Walsh understands that Cagan's and Friedman's definition is only approximately acceptable and only for small values of inflation. Walsh writes: „*For small values of the rate of inflation $\pi/(1 + \pi)$ is approximately equal to π , so [seigniorage] can be thought of as the product of a tax rate of π , the rate of inflation, and a tax base of h , the real stock of base money.*“ (Walsh, 2010, p. 139). Easterly, Mauro and Schmidt-Hebbel say „*The estimates [of the revenue maximising rate of inflation] using the Cagan form also usually define the inflation rate affecting money demand as the percent change in prices...when theory implies that the correct opportunity cost of holding money per period is the inflation rate divided by one plus the inflation rate.*“ (Easterly et al., 1995, p584)

3. A correct definition of seigniorage

After examining an error repeated in many papers and textbooks, we can propose a correct definition of monetary seigniorage, a formula, and a graph of it.

Seigniorage or inflation tax is a tax imposed on the holders of money balances by depreciating their value by printing more money. It is the product of a tax base, which is the monetary base, and a tax rate, which is the rate of depreciation of money.

$$S = MB \cdot \frac{\mu}{1+\mu} \quad (3)$$

where MB is the monetary base and μ is the growth rate of the monetary base.

In a steady state, we can substitute the rate of change of the monetary base with the rate of inflation because, in a steady state, the two variables are identical. It is however correct to use the growth of the monetary base μ in the formula as the general rule.

Using the growth of the monetary base instead of the rate of inflation in the formula of seigniorage makes sense even in a growing economy where an increase in the monetary base does not necessarily translate into inflation.

If under economic growth, an increase in the monetary base is absorbed by the growing volume of transactions, and the price level does not change, the government still appropriates resources from the public by preventing the appreciation of money which would otherwise take place under deflation. If the central bank did not print additional money, the price level would fall, and people could buy more for their money. By preventing deflation, the central bank takes wealth from people in the same way as it takes it through inflation under a steady state.

It is worth noting that in Keynes's example, the word "inflation" refers, as was common at that time, to a change in the monetary base rather than to a change in the price level.

Now define the ratio of seigniorage to GDP

$$s = \frac{\Delta MB}{GDP}$$

Assume nominal GDP equalled $P \cdot Y = M \cdot V = (MB \cdot m) \cdot V$ before the increase in the monetary base when the monetary base was MB . The new nominal GDP equals $((1 + \mu) \cdot MB \cdot m) \cdot V$ after the monetary base was increased by $\mu \cdot MB$ to $(1 + \mu) \cdot MB$.

Hence, seigniorage as a percentage of GDP can be expressed as

$$s = \frac{\Delta MB}{GDP} = \frac{\mu \cdot MB}{(1 + \mu) \cdot MB \cdot m \cdot V}$$

We can define ratio k as the monetary base to nominal GDP

$$k = \frac{MB}{MB \cdot m \cdot V} = \frac{1}{m \cdot V}$$

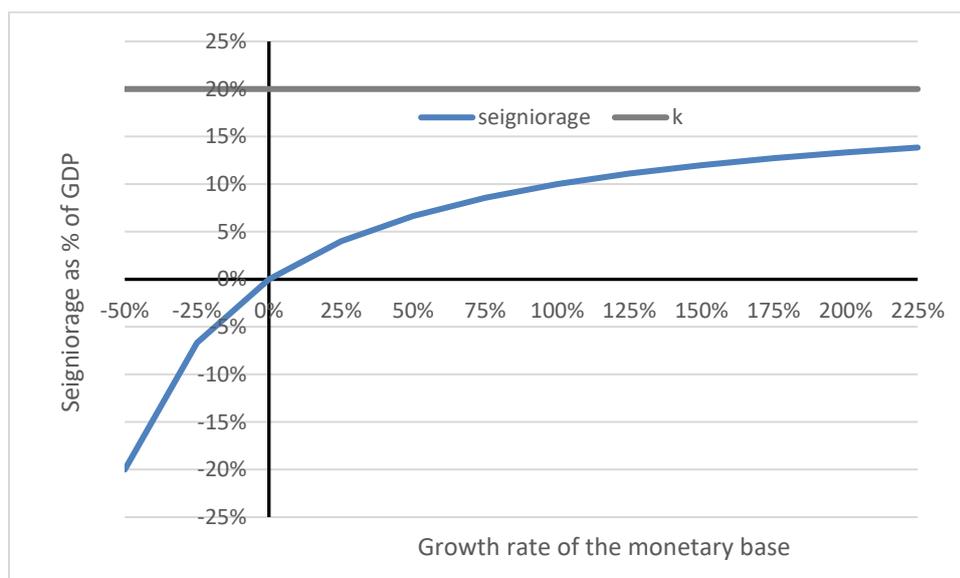
hence seigniorage as a percentage of GDP can be defined as follows:

$$s = k \cdot \frac{\mu}{1 + \mu} \quad (4)$$

where k is constant in a steady state.

Therefore, seigniorage as a function of μ is a hyperbola asymptotically approaching to k as illustrated in Fig. 2.

Fig. 2: The Curve of Seigniorage

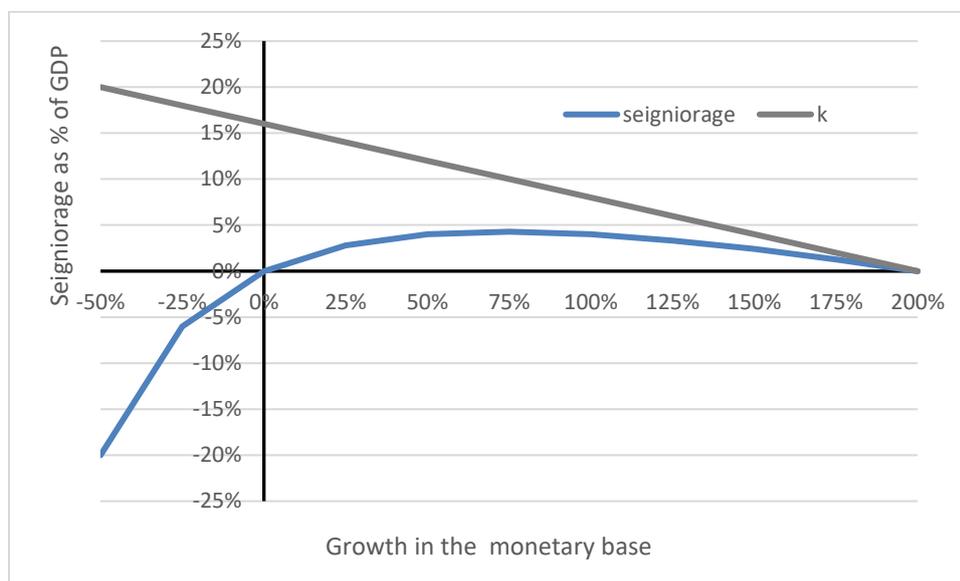


The money growth can be negative or positive in the interval $(-1, \infty)$. Whatever high the growth of the monetary base is, the seigniorage can never be higher than coefficient k , the ratio of the monetary base to GDP.

Seigniorage is negative if the central bank decreases the monetary base. For instance, if the central bank sells gold reserves or foreign exchange, it gives up some wealth. The sale of assets by the central bank means the withdrawal of money from circulation so the money stock decreases. This causes deflation, and wealth is thus transferred from the central bank to the public which enjoys an appreciation of money balances.

With high rates of inflation, people tend to use barter or another currency instead of official legal tender. As a consequence, the velocity of circulation will increase, and k will not be constant. The coefficient k will decrease. Probably, the higher the rate of inflation is the lower k will be. Under hyperinflation the curve of seigniorage will bend down, making “the Laffer curve of seigniorage” to use an analogy with the Laffer curve used for explicit taxes, as illustrated in Fig. 3.

Fig. 3: The Laffer curve of seigniorage



4. Discussion

This article revealed an error rooted in the literature on seigniorage. The authors using the mathematically wrong formula might have used it deliberately as a simplification (it produces relatively small errors for small rates of inflation) or by mistake. Anyway, the wrong formula has then been rooted down in economic textbooks.

This paper has clarified the concept of seigniorage. The unambiguous formula presented in this paper should be consequently used in textbooks and articles instead of the misleading formula used by most authors.

The proposed formula can be also used for the calculation of the relative size of the seigniorage of individual countries easily out of their monetary base growth rates.

5. Conclusion

Seigniorage or inflation tax is an important concept in economics. An error that occurred in the theory of seigniorage should be retrieved. Seigniorage should be contained in macroeconomics textbooks as well as in monetary theory and public

finance textbooks in its right form as the product of the monetary base and the rate of the monetary base growth divided by the rate of the monetary base growth plus one.

In this article, a curve of seigniorage was proposed, which is a hyperbole asymptotically approaching the ratio of the monetary base to nominal GDP. The formula and the curve of seigniorage clearly explain that the government can never appropriate more than the ratio of the monetary base to the nominal GDP, which is usually a few percentages of GDP. The knowledge of this property of seigniorage could deter governments from resorting to hyperinflations.

References

- Bailey, M. J., 1956. "The Welfare Cost of Inflationary Finance". *Journal of Political Economy*, Volume 64, N. 2 (April 1956). (<https://www.jstor.org/stable/1826826>)
- Blanchard, O., 1997. "Macroeconomics". Prentice-Hall, Inc. New Jersey.
- Bofinger, P., 2001. "Monetary Policy". Oxford University Press, New York.
- Cagan, P., 1956. "Monetary Dynamics of Hyperinflation". In: Friedman, M. (ed.): *Studies in the Quantity Theory of Money*, University of Chicago Press. (<http://people.bu.edu/rking/SZGcourse/Cagan.pdf>)
- Čermáková, K.; Filho, E.A.H. Effects of Expansionary Monetary Policy on Agricultural Commodities Market. *Sustainability* **2021**, *13*, 9317. <https://doi.org/10.3390/su13169317>
- Easterly, Mauro, Schmidt-Hebbel, 1995, "Money Demand and Seigniorage-Maximizing Inflation". *Journal of Money, Credit, and Banking*, 27, 1995.
- Friedman, M., 1953. "Discussion of the Inflationary Gap". In: *Essays in Positive Economics*. The University of Chicago Press. Chicago & London.
- Friedman, M., 1971. "Government Revenue from Inflation". *Journal of Political Economy*, Vol. 79, N. 4 (July—August 1971) (<http://debis.deu.edu.tr/userweb/yesim.kustepeli/dosyalar/friedman1971.pdf>)
- Keynes, J. M., 1924. "A Tract on Monetary Reform". London, Macmillan. (<https://delong.typepad.com/keynes-1923-a-tract-on-monetary-reform.pdf>)
- Klein, M., Neumann, M.: "Seigniorage: What Is It and Who Gets It?". *Weltwirtschaftliches Archiv*, Bd. 126, H. 2 (1990), pp. 205-221
- Mankiw, G., 1987. "The Optimal Collection of Seigniorage: Theory and Evidence". *Journal of Monetary Economics*, 20, September 1987.
- Mankiw, G., 2003. "Macroeconomics", fifth edition. Worth Publishers, New York, 2003
- Walsh, C., 2010. "Monetary Theory and Policy", third edition. MIT, <http://www2.um.edu.uy/dtrupkin/walsh.pdf>