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Economic Impacts of an Emissions Trading Scheme Pilot in Oligopolistic Agri-Food Supply Chains: A Network Equilibrium Analysis

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ABSTRACT

Emissions Trading Scheme (ETS) pilot programs impose binding quota constraints and enable allowance trading, reshaping cost structures and strategic interactions in oligopolistic agri-food supply chains. This paper quantifies the resulting economic impacts, including equilibrium prices, profits, and trade flows, by developing a multi-tier network equilibrium model that links upstream suppliers, downstream manufacturers, domestic and international demand markets, and a carbon trading center under an Emissions Trading Scheme pilot setting. Suppliers invest in low-carbon technologies, while manufacturers undertake labor-efficiency investments that affect unit costs and throughput, with proximity-based spillovers captured via a grid-distance mechanism. The equilibrium conditions are formulated as a variational inequality framework and computed numerically, enabling systematic comparative statics analysis under alternative quota stringency and trading conditions. Using China-EU garlic trade as an illustrative case, the numerical analysis indicates that tighter policy constraints and trading conditions shift production and allowance-trading patterns, with corresponding changes in prices, profits, and emissions across tiers. It also shows that moderate efficiency investment can improve productivity and may reduce aggregate emissions, whereas very high unilateral investment tends to exhibit diminishing returns and can be associated with non-smooth adjustments in network allocations. Finally, coordinated upstream-downstream investment is generally associated with more stable outcomes than isolated initiatives. The framework offers a decision-relevant tool for evaluating Emissions Trading Scheme pilot designs in regulated international agri-food trade networks.

1. Introduction

Since 2000, the average level of greenhouse gas emissions in the agrifood system has hovered within a persistently high range and has comprised about 25-30 percent of total greenhouse gas

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emissions. The FAO dataset shows that the level of greenhouse gas emissions from the agrifood system increased from about 14.7 GtCO2-eq in 2000 to about 16.5 GtCO2-eq in 2019 and continued to hold steady in 2022 at about 16.2 GtCO2-eq with no evidence [1]. Moreover, the contribution of regional agriculture sectors is uneven, with Asia accounting for the largest share [2]. These patterns suggest that effective mitigation policies targeting agrifood production and trade are essential for achieving global decarbonization goals.

In international agri-food production and trade, key commodity sectors are increasingly dominated by a small number of large transnational firms, implying oligopolistic market structures. Commodities (e.g., garlic, cotton, and soybeans) are produced through large-scale mechanized systems that rely heavily on chemical inputs and operate via long supply chains, which can increase the product-level carbon footprint relative to smaller, decentralized operations. Empirical evidence suggests that, while scaling may improve certain efficiency measures, it may also be associated with higher carbon footprints in specific production contexts [3, 4]. Thus, focusing on oligopolistic agri-food firms is practically important. These actors contribute materially to sectoral emissions and, due to their scale, may also deliver sizable emission reductions when incentivized or regulated.

Notwithstanding the importance of mitigating emissions in the agrifood system, carbon regulation for agricultural production and related supply-chain activities remains relatively limited in many jurisdictions, including the European Union and China, where carbon-market participation in farming is still largely voluntary and small in scale [5]. This regulatory gap is particularly salient in international agri-food trade dominated by a few large firms, because emissions embedded in production and cross-border logistics can accumulate across tiers. Against this background, introducing pilot Emissions Trading Scheme programs that extend carbon-market incentives and constraints to major agrifood enterprises, and their supply chains represents a pragmatic policy approach to advancing mitigation objectives in the agrifood sector.

From the perspective of oligopolistic agri-food enterprises, China's agri-food exports to the European Union are increasingly shaped by tightening decarbonization and import-related policy requirements. These include enhanced product-level carbon information and sustainability compliance initiatives (e.g., footprint disclosure frameworks and rules under the Farm-to-Fork strategy), as well as policy expectations regarding carbon-related measures applied to imports [6-8]. As a result, Chinese export-oriented oligopolistic firms may face rising compliance costs and stricter constraints on embedded emissions along the supply chain. In parallel, as discussed above, the introduction of pilot Emissions Trading Scheme programs can further strengthen carbon-related incentives and constraints at the production side. Taken together, these forces imply a “double pressure” on supply-chain participants, making it important to quantify the associated economic impacts, such as changes in equilibrium prices, profits, and trade flows in China-EU international agri-food supply chains.

Motivated by the above context, we develop a multi-level oligopolistic international agricultural supply chain network model that captures the interactions among suppliers, manufacturers, domestic and international demand markets, and a carbon trading center under a pilot Emissions Trading Scheme setting. While prior studies have examined carbon regulation, carbon trading, and investment activities in agri-food settings, the literature still lacks an integrated equilibrium framework that simultaneously: (i) quantifies market-equilibrium outcomes when pilot Emissions Trading Scheme policies are introduced into agri-food supply chains, (ii) evaluates supply-chain economic performance under the competing objectives of mitigation responsibilities and oligopolistic firms' operational incentives, and (iii) incorporates the joint role of low-carbon technology investment and labor-efficiency investment, which can reshape costs and thereby affect policy transmission along the network. To address this gap, we formulate the equilibrium conditions and express them in a variational inequality (VI) framework, which enables systematic analysis of strategic decisions and policy constraints as reflected in equilibrium prices, profits, and trade flows. The proposed framework is implemented computationally and illustrated through a numerical study motivated by the China-EU garlic export supply chain.

The organizational form of our research is as follows: In Section 2, the introduction to the literature review and the paper contributions is placed. In Section 3, an international agricultural supply chain oligopoly model with multi-level factors such as carbon quota trading activity, low-carbon technology investment activity, and manufacturing labor efficiency investment activity is set up. In Section 4, the qualitative properties are analyzed. In Section 5, algorithms are designed to solve the model. In Section 6, supply chain entities' decision-making behaviors are analyzed from different perspectives in numerical simulation cases with the Chinese garlic export supply chain to the EU within the sustainable policy environment in the EU context. The results are summed up in Section 7.

2. Literature Review and Contributions

This section reviews the literature related to our oligopolistic international agri-food supply-chain network equilibrium model under Emissions Trading Scheme pilot regulation. Specifically, we organize the review into four streams: (i) Emissions Trading Scheme regulation in agri-food supply chains, (ii) low-carbon technology investment in agri-food supply chains, (iii) labor-efficiency investment in agri-food supply chains, and (iv) investment spillovers and grid-distance representations in agri-food supply chains. Section 2.5 then summarizes the resulting research gaps and positions the contributions of this paper.

2.1 Emissions Trading Scheme Regulation in Agri-food Supply Chains

The Emissions Trading System (ETS) is a market-based regulatory instrument that sets an aggregate cap on greenhouse gas emissions and allocates emission allowances to regulated entities, which can be traded to generate a carbon price signal and achieve mitigation at lower cost [9]. The ETS sets the regulatory and operational environment within which emission regulation activities occur; trading in carbon allowances is the market itself that carries out buying and selling activities [10]. However, in most existing ETS frameworks, agricultural production and many agri-food supply-chain activities remain outside formal coverage and are instead primarily addressed through non-ETS policies or voluntary carbon-market initiatives, including in the European Union and China. Although carbon trading is a broad term covering both compliance and voluntary markets, this study focuses on the compliance type cap-and-trade mechanism embodied in ETS pilots. Accordingly, we review related supply-chain studies that model carbon trading under carbon constraints as the closest analytical basis for understanding ETS-type regulation in agri-food supply chains.

Recent studies have increasingly embedded ETS-type cap-and-trade mechanisms into supply-chain models to examine how carbon caps and tradable allowances reshape operational decisions and economic outcomes. A recent systematic review synthesizes the fast-growing “supply chain management under cap-and-trade regulation” literature and highlights how carbon caps and trading signals have been incorporated into decision models to evaluate impacts on costs, pricing, production, and coordination outcomes, while also identifying unresolved challenges in linking policy design with market-equilibrium responses [11]. Building on this stream, recent agri-food‐focused studies have started to examine cap-induced decisions under operational uncertainties. For instance, one study investigates a carbon-capped food supply chain with demand fluctuations and compares alternative contract types, showing that contracting choices can materially alter incentives for green investment and the distribution of profits under emission caps [12]. Complementarily, another study analyzes a food supplier's equilibrium carbon-abatement investment in a supply-chain network and evaluates implications for profits, consumer surplus, and emissions under policy instruments such as subsidies, indicating that key cost parameters and network scale can shape equilibrium outcomes [13]. At the policy–sector interface, a recent contribution discusses how carbon pricing and emissions trading can affect food-system sustainability and equity, emphasizing trade-offs in policy effectiveness, distributional impacts, and implementation design [14]. In addition, optimization-based cap-and-trade formulations continue to expand in related supply-chain contexts, providing methodological references for incorporating carbon caps and trading signals into network-level decision models [15].

Despite these advances, clear gaps remain for ETS-type cap-and-trade analysis in agri-food supply chains. Existing studies largely emphasize contract design or centralized optimization in single-chain settings, with limited attention to predictive network equilibrium analysis in international agri-food trade where market power and strategic interactions matter. Moreover, multi-tier integration that links policy constraints and investment or efficiency adjustments to equilibrium prices, profits, and trade flows remains underdeveloped, motivating our variational-inequality-based network equilibrium approach.

2.2 Low-carbon Technology Investment in Agri-food Supply Chains

Low-carbon technology investment in agri-food supply chains refers to firms' capital allocation to adopt, develop, or implement emission-reducing technologies and practices across production and logistics activities, with the aim of lowering carbon intensity or total emissions per unit of output. Such emission reduction is typically achieved through technology adoption and therefore requires non-negligible investment costs, which in turn interact with operational decisions under carbon-related constraints [16]. Accordingly, recent supply-chain studies commonly model low-carbon technology investment as a strategic decision variable that affects demand, costs, and performance.

One stream of research emphasizes how governance structures and contractual arrangements shape investment incentives. Different channel structures affect both the adoption level and the profitability of low-carbon technologies [17, 18], and recent evidence further highlights that contract design and adoption can materially reshape firms' incentives for green technological innovation and low-carbon technology uptake [19].

Another line of work takes a policy-design perspective, examining how carbon-policy instruments influence firms' technology choices and the economic viability of emissions-reduction investments [20, 21]. Comparative analyses suggest that carbon-policy design affects the attractiveness of investing in higher-efficiency technologies, and that emissions-reduction investments in production and transportation may become economically viable under certain policy settings [22]. In closed-loop supply chain contexts, carbon prices and carbon taxes may exhibit a non-linear complementary relationship, implying that integrated strategies can improve both economic and environmental performance within certain parameter ranges [23].

Overall, prior studies clarify how contractual arrangements and carbon-policy instruments shape incentives for low-carbon technology investment and related operational adjustments in supply chains. However, existing research offers limited predictive network equilibrium analysis tailored to international agri-food supply chains with oligopolistic competition, where upstream investment and downstream efficiency adjustments jointly affect market outcomes. In particular, the interaction between low-carbon technology investment and labor-efficiency improvements, and its implications for equilibrium prices, profits, and trade flows under ETS-type carbon constraints, remains underexplored. This motivates the integrated network equilibrium framework developed in this paper.

2.3 Labor Efficiency Investment in Agri-food Supply Chains

Labor efficiency investment refers to firms' resource allocation aimed at improving effective labor productivity and operational responsiveness along supply chains [24]. In supply-chain settings, such investments can take the form of targeted workforce capability enhancement and process enabled improvements, which reduce effective unit operating costs and strengthen resilience to demand shocks. In this study, labor efficiency investment is modeled as an efficiency-enhancing decision that affects unit costs and throughput in the agri-food supply-chain network, thereby shaping equilibrium outcomes under carbon-related constraints. .

Recent studies have increasingly examined labor efficiency investment from an operational and resilience perspective, especially in labor-intensive sectors such as agriculture. In perishable-food supply chains exposed to pandemic-related disruptions, models incorporating labor availability in production, storage, and distribution have been developed to assess the role of labor constraints in disruption propagation and recovery [25]. Complementary research has proposed multi-period supply chain network frameworks that evaluate productivity-oriented investments (e.g., safety and health measures) using NPV-based criteria, showing that productivity investment can be beneficial to both employees and businesses under certain demand/consumer settings [26]. Related game-theoretic results further support that productivity-oriented investment can generate productivity gains and welfare improvements for supply chain stakeholders [27, 28]. In our setting, such productivity-oriented investments are abstracted as labor-efficiency improvements that reduce effective unit operating costs and enhance throughput in production, storage, and distribution.

Under carbon-related constraints, labor-efficiency improvements can interact with carbon trading incentives by altering production costs and capacity utilization, thereby affecting allowance demand and compliance exposure. In parallel, carbon quota trading schemes have been shown to influence investment behavior and supply chain configuration. Empirical evidence suggests that carbon emission trading can curb over-investment and improve investment efficiency, with stronger effects observed in firms with stronger corporate governance [29]. From a network-design viewpoint, integrating carbon trade credits with trade credit mechanisms in eco-efficiency models can improve profitability, and supply chain outcomes may be highly sensitive to carbon trading conditions [30].

In summary, existing studies show that labor-efficiency investment can enhance supply-chain productivity and resilience, and that carbon trading mechanisms may reshape investment incentives and network configuration. However, predictive equilibrium analysis that jointly links labor-efficiency improvements and carbon-trading constraints to equilibrium prices, profits, and trade flows in oligopolistic international agri-food supply chains remains limited, motivating our model formulation.

2.4 Spillover Effect and Grid Distance in Agri-food Supply Chains

The supply chain investment spillover effect was defined as the positive externality that the non-investing entities enjoyed without direct reciprocation as a result of core firms' investment in productivity-enhancing activities such as the adoption of new technology, training, and process improvements [31]. The history of classical research shows that R&D and technology investments result in regional knowledge spillovers that favor other firms within the environment in terms of patent citations, labor mobility, and learning among suppliers [32, 33].

Grid distance provides an operational way to translate geographic proximity into a measurable decay pattern for investment spillovers across supply-chain entities. In practice, it can be computed as the Euclidean distance between the locations of firms and then embedded into a distance‑decay weighting scheme, so that spillover intensity decreases as spatial separation increases [34]. Moreover, supply-chain diffusion and spillover processes can exhibit strong geographic attenuation, where the influence transmitted through proximity diminishes rapidly beyond a spatial boundary, motivating an explicit distance-decay structure when modeling spillovers jointly through space and supply-chain connections [35]. Therefore, introducing grid-distance-based decay enables our model to parsimoniously capture the spatially dependent strength of investment spillovers while maintaining a transparent link between geography and inter-firm externalities.

Recent agri-food research has increasingly documented that proximity-conditioned spillovers are empirically relevant in food-related supply-chain systems. For example, spatial analyses of cold-chain logistics for fresh agri-products reveal significant geographic dependence in carbon-emission outcomes, indicating that logistics-related externalities can transmit across neighboring regions rather than remaining locally contained [36]. Complementary evidence further suggests that food-system resilience drivers may generate measurable spatial spillover effects with identifiable attenuation patterns, implying that proximity can shape the reach and strength of such externalities across regions [37]. At the industry level, studies on technological innovation in food production also report spatial spillovers in resilience and performance, reinforcing the view that technology- and investment-driven benefits can diffuse beyond the investing entity through geographically proximate interactions [38]. However, these insights are rarely integrated into a unified network equilibrium setting that explicitly embeds distance-decayed investment spillovers within multi-tier international agri-food supply chains under ETS-type carbon constraints, motivating our modeling approach.

Overall, existing studies indicate that agri-food supply-chain spillovers are often proximity-conditioned and can attenuate with distance. However, these insights have seldom been integrated into a predictive network equilibrium setting that explicitly parameterizes distance-decayed investment spillovers and evaluates their implications for equilibrium prices, profits, and trade flows under ETS-type carbon constraints, motivating our modeling approach.

2.5 Contributions

In contrast to the existing research, the originality of our paper has the following novel contributions:

1. We develop a multi-tier network equilibrium (VI) model for an international agri-food supply chain under an ETS-type cap-and-trade regime, explicitly linking quota-based compliance to equilibrium prices, profits, emissions, and trade flows while accounting for product perishability.
2. We characterize how market power in oligopolistic agri-food trade interacts with carbon regulation by quantifying the incidence of policy stringency, showing how changes in quota tightness and trading conditions are transmitted to firms' pricing or production decisions and cross-border shipments.
3. We incorporate labor-efficiency investment and distance-decayed spillovers into the equilibrium framework and embed them into operational decisions throughput and unit-cost adjustments, enabling the analysis of coordination versus unilateral investment and their implications for economic outcomes and emission performance across tiers.
4. Using the China-EU garlic export supply chain as an illustrative case, we provide numerical experiments that demonstrate the model's tractability and generate decision-relevant insights on how carbon-policy stringency and efficiency requirements jointly shape market outcomes.

3. The Oligopolistic International Agricultural Supply Chain Network Model with Low-Carbon Technology Investment, Labor Efficiency Investment, and Carbon Quota Trading

3.1 Research Objectives and Hypotheses

3.1.1 Research objectives

Our study aims to pursue three objectives:

(O1) To develop a multi-tier network equilibrium (VI) framework for an oligopolistic international agri-food supply chain with a carbon trading center under an ETS-type cap-and-trade regime, and to characterize how compliance requirements map into equilibrium prices, profits, trade flows, and emissions across tiers.

(O2) To quantify how changes in policy stringency and trading conditions (e.g., tighter effective quota availability and alternative trading environments) are transmitted through the network, reshaping firms' production, shipment, and allowance-trading decisions, and thereby the distribution of economic outcomes between upstream suppliers, downstream manufacturers, and domestic or foreign demand markets.

(O3) To examine the joint roles of low-carbon technology investment and labor-efficiency investment, together with grid-distance-mediated spillovers, in determining equilibrium responses, specifically, how these investment mechanisms interact with carbon constraints to alter unit costs and throughput, and ultimately the equilibrium patterns of prices, profits, trade flows, and emissions.

3.1.2 Hypotheses

To guide the equilibrium and numerical analyses, we state the following testable hypotheses.

H1. Under an ETS-type cap-and-trade regime, a tighter effective quota constraint or a less favorable trading environment that increases compliance pressure is expected to raise firms' marginal compliance costs and thereby shift equilibrium decisions, leading to systematic changes in equilibrium prices, profits, allowance-trading volumes, and cross-border trade flows.

H2. Stricter target-market sustainability requirements are expected to strengthen firms' incentives to adjust technology and operational choices to maintain market access, resulting in reallocated production or shipment patterns and measurable impacts on equilibrium prices, profits, trade flows, and emissions in the international agri-food supply chain.

H3. Low-carbon technology investment by upstream suppliers and labor-efficiency investment by downstream manufacturers are expected to jointly shape equilibrium outcomes through their effects on unit costs, throughput, and emissions; moreover, grid-distance-mediated spillovers are expected to amplify the marginal impact of investment for geographically proximate entities, implying heterogeneity in equilibrium responses across tiers and locations.

3.2 Basic Assumptions

The oligopolistic international agri-food supply chain considered in this paper is modeled as a multi-tier network consisting of domestic upstream suppliers, domestic downstream manufacturers, domestic/foreign demand markets, and one domestic carbon-trading center (see Figure 1). Specifically. it is assumed that there are I domestic suppliers, J domestic manufacturers, K foreign demand markets, and one domestic carbon trading center (see Figure 1); $I, J, K \in \mathbb{N}$.

Each supplier is divided into a distribution department and a production department. The production department of each supplier focuses on implementing low-carbon technology investments to reduce carbon emissions. Each manufacturer is divided into a distribution department and multiple production departments. Each manufacturer's production department focuses on labor efficiency investments aimed at improving production processes that depend on different demand countries. A typical production department of a manufacturer is denoted as $n=1,\cdots, N$.

Suppliers represent farming or primary-handling enterprises that provide semifinished products to manufacturers; manufacturers represent processors or exporters that supply processed products to domestic/foreign demand markets under direct export. Suppliers (oligopolistic agricultural enterprises) are classified as high-emitting members because fertilization, irrigation, and harvesting generate significant carbon emissions. However, manufacturers (agricultural product processors and exporters) are classified as low-emitting members because processing and packaging involve a comparatively lower level of carbon emissions.

We further assume heterogeneous emission intensities across tiers: suppliers are relatively high-carbon emitters (e.g., due to farming inputs and field operations), whereas manufacturers are relatively low-carbon emitters (e.g., processing and packaging). Under the ETS-pilot setting, if a member's emissions exceed its allocated allowances, it purchases additional allowances from the carbon trading center; if it holds surplus allowances, it can sell them to the carbon trading center.

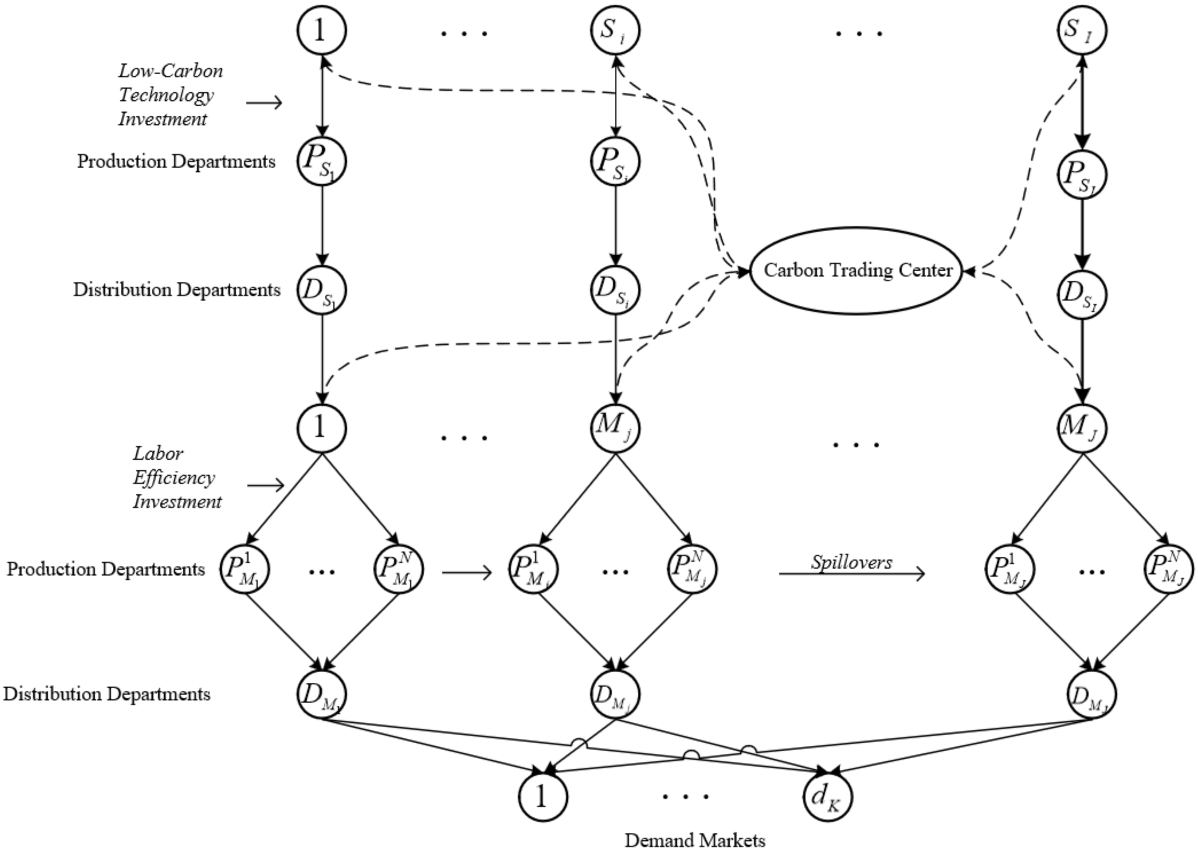

Fig. 1. Oligopolistic international agricultural supply chain network with low-carbon technology and labor efficiency investment under ETS pilot (I, J,K,N)

To identify the network members concisely and clearly, we introduce some notation. A typical supplier is denoted by S_i, a typical manufacturer is denoted by M_j, and a typical demand market is denoted by d_k ; $i \in \{1,\cdots,I\}$, $j \in \{1,\cdots,J\}$, $k \in \{1,\cdots,K\}$. Moreover, notations for key parameters and functions are listed in Table 1.

Table 1
Notation for the international supply chain network model

Notation	Definition
$q_{S_iM_j}$	The transaction volume of semifinished agricultural products between supplier S_i and manufacturer M_j, with all such volumes grouped into a column vector $\boldsymbol{Q}^1 \in \Re_+^{IJ}$.
$q_{M_jP_{M_j}^n d_k}$	The transaction volume of processed agricultural products between production department $P_{M_j}^n$ of manufacturer M_j and demand market d_k, with all such volumes grouped into a column vector $\boldsymbol{Q}^2 \in \Re_+^{JNK}$.
ρ_{S_i}	The retail price of products supplied by supplier S_i.
ρ_{M_j}	The retail price of products supplied by manufacturer M_j.
P_{d_k}	The demand price in demand market d_k.
s	The low-carbon technology investment level for supplier S_i, where 0<s<1.
l_{M_j}	The available labor of manufacturer M_j.
$v_{M_jP_{M_j}^n}$	The investment of production department $P_{M_j}^n$ of manufacturer M_j in labor efficiency, with all such volumes grouped into a column vector $\boldsymbol{v} \in \Re_+^{JNK}$.
$\overline{v}_{M_jP_{M_j}^n}$	The upper bound of investment in labor efficiency.
w_{M_j}	The hourly wage paid for a unit of labor in manufacturer M_j section.
η_{M_j}	The productivity of manufacturer M_j, expressed as a function of $v_{M_jP_{M_j}^n}$.
$e_n^{spillover}$	The labor efficiency investment spillover effect received by the manufacturer M_j's production department $P_{M_j}^n$, $n = 1,...,N$
τ	The tariff rate imposed on exported processed products.
cap_{S_i}	The carbon emission quota allocated free of charge by the domestic government to S_i.
cap_{M_j}	The carbon emission quota allocated free of charge by the domestic government to M_j.
ε	The unit carbon trading fee charged by the domestic carbon trading center for transactions with suppliers and manufacturers.
ω	The unit carbon trading price between domestic suppliers and manufacturers.
α_{S_i}	The unit carbon emission coefficient for products produced byr S_i.
α_{M_j}	The unit carbon emission coefficient for products produced domestically by M_j.
$c_{S_iM_j}$	The transaction cost of products provided by S_i, expressed as a function of $q_{S_jM_j}$.
$c_{M_jd_k}$	The transaction cost of products provided by M_j, expressed as a function of $q_{M_jP_{M_j}^n d_k}$.
c_{d_k}	Unit transaction cost incurred by consumers in demand market k ,expressed as a function of $q_{M_jP_{M_j}^n d_k}$
$c_{S_i}^t$	The transaction cost incurred by the carbon trading center when conducting carbon trading with S_i.
$c_{M_j}^t$	The transaction cost incurred by the carbon trading center when conducting carbon trading with M_j.
$c_{M_jP_{M_j}^n D_k}^p$	The tansportation cost of production department $P_{M_j}^n$ of manufacture M_j.
$c_{S_i}^g$	The cost of low-carbon technology investment made by the supplier to reduce carbon emissions.
$f_{S_iM_j}$	The production cost of supplier i, expressed as a function of $q_{S_jM_j}$.
$f_{M_jP_{M_j}^n d_k}$	The production cost of production department n of manufacturer j, expressed as a function of $q_{M_jd_k}$.
d_k	The demand quantity in demand market k.
θ	The cost coefficient for low-carbon technology investment for supplier S_i.
δ	The coefficient indicating the impact of low-carbon investment on production, where δ >0.
x	The initial labor input, where x >0.

Notation	Definition
y	The coefficient for impact produced due to investment in labor efficiency, where y >0.
o_{M_j}	The coefficient that relates wages to labor availability, where o_{M_j} >0.
w_{M_j}	The hourly wage, where w_{M_j} >0.
φ	The spillover effect coefficient, where φ >0.
β	The grid distance coefficient, where β >0.
ϖ	The scale effect coefficient, where ϖ >0.
μ	The perishability coefficient, where 0< μ <1.

Given the above assumptions, we next model suppliers' decisions (Section 3.3), manufacturers' decisions (Section 3.4), carbon trading center's decisions (Section 3.4), and then integrate them into a VI equilibrium.

3.3 The Behavior of the Suppliers and Their Optimal Conditions

In this section, we characterize suppliers' optimal decisions in an oligopolistic agri-food supply chain. Each supplier consists of a production unit that undertakes low-carbon technology investment and a distribution unit that trades with manufacturers.

A supplier's decision problem includes production costs, transaction costs, technology-investment costs, and ETS-related compliance costs. Production costs are associated with total output q_{S_i} and are specified as $f_{S_iM_j} = f_{S_iM_j}\left(Q^1\right)$, where Q^1 collects all suppliers' outputs, thereby capturing oligopolistic interactions through output-dependent marginal costs.

Supplier-manufacturer transactions incur per-unit frictions captured by the transaction cost function $c_{S_iM_j} = c_{S_iM_j}\left(q_{S_iM_j}\right)$. At equilibrium, each supplier's total production equals its total shipments to manufacturers, yielding the flow-balance condition in equation (1):

$$q_{S_i} = \sum_{j=1}^{J} q_{S_iM_j} \tag{1}$$

The flow conservation equation (1) indicates that the total production volume of a supplier is equal to the total transaction volume between the supplier and the manufacturers.

Suppliers choose a low-carbon technology level $S_i \in (0,1)$, incurring the convex investment cost:

$$c_{S_i}^{g} = \theta \frac{s^2}{2} \tag{2}$$

Under the ETS-type cap-and-trade regime, suppliers face compliance costs when emissions exceed their allocated allowances cap_{S_i}. Let the supplier's baseline emission intensity be α_{S_i}; emissions generated by production are $\alpha_{S_i} q_{S_i}$, and they can be expressed as $\alpha_{S_i}\sum_{j=1}^{J} q_{S_iM_j}$. We treat the carbon trading price faced by suppliers as exogenous in the baseline model. Accordingly, the supplier's allowance shortfall is $\alpha_{S_i}\sum_{j=1}^{J} q_{S_iM_j} - cap_{S_i}$.

The effectiveness of low-carbon technology investments is reflected in the reduction in carbon emissions [39]. The reduced emissions of S_i are given by:

$$\alpha_{S_i}\left(1-s\right)\sum_{j=1}^{J} q_{S_iM_j} \tag{3}$$

Thus, the amount a typical supplier S_i must pay to the carbon trading center to purchase additional carbon emission quotas is as follows:

$$(\varepsilon+\omega)\left(\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}\right) \tag{4}$$

In addition to reducing emissions, low-carbon technology investment may also improve production efficiency. To parsimoniously incorporate this co-benefit, we scale the supplier's effective output by $(1+\delta s)$, where $\delta>0$ measures the productivity gain associated with low-carbon investment, as shown below:

$$q_{S_i}^{adjusted}=(1+\delta s)q_{S_i} \tag{5}$$

where $q_{S_i}^{adjusted}$ is the new production level.

In the supply chain network, the profit of S_i, π_i, can be expressed as:

$$\pi_{S_i}=(1+\delta s)\sum_{j=1}^{J}\rho_{S_i}q_{S_iM_j}\text{-}f_{S_iM_j}(\boldsymbol{Q}^1)-\sum_{j=1}^{J}c_{S_iM_j}(q_{S_iM_j})-\theta\frac{s^2}{2}-(\varepsilon+\omega)\left(\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right) \tag{6}$$

The objective function of the supplier is to maximize its profit, which can be expressed as follows:

$$\text{Max}\,\pi_{S_i}=(1+\delta s)\sum_{j=1}^{J}\rho_{S_i}q_{S_iM_j}-f_{S_iM_j}(\boldsymbol{Q}^1)-\sum_{j=1}^{J}c_{S_iM_j}(q_{S_iM_j})-\theta\frac{s^2}{2}-(\varepsilon+\omega)\left(\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right) \tag{7}$$

s.t.

$$q_{S_iM_j}\geq 0,\rho_{S_i}\geq 0,\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\geq 0. \tag{8}$$

To characterize the Nash equilibrium among competing suppliers under the above convexity and regularity conditions, we introduce the Lagrange multiplier λ_{S_i} associated with the quota-related constraint in (8), and write the corresponding Lagrangian function L_{S_i} for the supplier's profit, incorporating the carbon quota constraint, which can be formulated as:

$$\begin{aligned}L_{S_i}=&(1+\delta s)\sum_{j=1}^{J}\rho_{S_i}q_{S_iM_j}-f_{S_iM_j}(\boldsymbol{Q}^1)-\sum_{j=1}^{J}c_{S_iM_j}(q_{S_iM_j})-\theta\frac{s^2}{2}-(\varepsilon+\omega)\left(\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)\\&+\lambda_{S_i}\left(\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)\end{aligned} \tag{9}$$

Due to the noncooperative game among the competing suppliers [40], the Nash equilibrium conditions among these suppliers can be written in form of the following Variational Inequality Problem (VIP):

VIP 1. Determine vector $(\boldsymbol{Q}^{1*},\boldsymbol{\lambda}_S{}^*)$ such that

$$\begin{aligned}&\sum_{i=1}^{I}\sum_{j=1}^{J}\left(-(1+\delta s)\rho_{S_i}+\frac{f_{S_iM_j}(\boldsymbol{Q}^1)+c_{S_iM_j}(q_{S_iM_j})}{\partial q_{S_iM_j}}+\left((1-s)(\varepsilon+\omega)-\lambda_{S_i}{}^*\right)\alpha_{S_i}\right)\times\left(q_{S_iM_j}-q_{S_iM_j}{}^*\right)\\&+\sum_{i=1}^{I}\left(cap_{S_i}-\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}{}^*\right)\times\left(\lambda_{S_i}\text{-}\lambda_{S_i}{}^*\right)\geq 0\end{aligned} \tag{10}$$

where $\boldsymbol{\lambda}_S\triangleq(\lambda_{S_1},\lambda_{S_2},\cdots,\lambda_{S_I})^T$ is the column vector grouped by all Lagrange multipliers for suppliers.

By taking the first-order derivatives of Equation (9) with respect to $q_{S_iM_j}$ and λ_{S_i}, we obtain the following optimality conditions:

$$\frac{\partial L_{S_i}}{\partial q_{S_iM_j}}=-(1+\delta s)\rho_{S_i}+\frac{f_{S_iM_j}(\boldsymbol{Q}^1)+c_{S_iM_j}(q_{S_iM_j})}{\partial q_{S_iM_j}}+\left((1-s)(\varepsilon+\omega)-\lambda_{S_i}{}^*\right)\alpha_{S_i}=0 \tag{11}$$

$$\frac{\partial L_{S_i}}{\partial \lambda_{S_i}} = cap_{S_i} - \alpha_{S_i}(1-s)\sum_{j=1}^{J} q_{S_i M_j}{}^{*} = 0 \tag{12}$$

Under standard differentiability and convexity assumptions on the cost components, these KKT and complementarity conditions provide a well-defined equilibrium characterization for the competing suppliers. Following the standard VI reformulation of convex Nash equilibrium problems [40], we express the suppliers' Nash outcome equivalently by the variational inequality in (10), which will be embedded into the integrated network equilibrium model.

3.4 Behavior of the Manufacturers and Their Optimal Conditions

In the international agricultural supply chain network, we consider J competing manufacturers that process upstream products and sell homogeneous processed goods to the demand-market tier. Each competing manufacturer has N production departments.

We specify manufacturers' decisions through the following cost components and operational constraints.

Manufacturers incur procurement costs when purchasing semi-finished products from suppliers; for M_j, the total procurement cost is given by (13):

$$\sum_{i=1}^{I} \rho_{S_i} q_{S_i M_j} \tag{13}$$

Each production department incurs production costs associated with shipment volumes to demand markets and labor-efficiency investment; the production cost of M_j is formulated as (14):

$$f_{M_j P_{M_j}^n d_k} = f_{M_j P_{M_j}^n d_k}\left(q_{M_j P_{M_j}^n d_k}, v_{M_j P_{M_j}^n}\right) \tag{14}$$

In addition, product transaction costs of M_j are incurred in manufacturer-market exchanges and are specified by (15):

$$c_{M_j d_k} = c_{M_j d_k}\left(q_{M_j P_{M_j}^n d_k}\right) \tag{15}$$

For foreign demand markets, exported products are subject to ad valorem tariffs, yielding the tariff cost of M_j in (16):

$$\tau \sum_{k=1}^{K} q_{M_j P_{M_j}^n d_k} \tag{16}$$

Manufacturers also incur transportation costs for shipments to demand markets; the corresponding cost function of M_j is given by (17):

$$c_{M_j P_{M_j}^n d_k}^{p} = c_{M_j P_{M_j}^n d_k}^{p}\left(q_{M_j P_{M_j}^n d_k}\right) \tag{17}$$

Because manufacturers are assumed to be low-emission firms, their realized emissions do not exceed the allocated allowances; hence they may sell surplus permits through the carbon trading center, yielding the revenue in (18):

$$(\omega - \varepsilon)\left(cap_{M_j} - \alpha_{M_j}\sum_{n=1}^{N}\sum_{k=1}^{K} q_{M_j P_{M_j}^n d_k}\right) \tag{18}$$

Furthermore, the product output of the manufacturer's production department is constrained by the labor input multiplied by the labor efficiency η_{M_j}. Unlike earlier studies where productivity was considered constant [26, 28], it is now modeled as a function of the labor efficiency investment made to enhance labor efficiency:

$$q_{M_jP_{M_j}^n d_k} \le \eta_{M_j} l_{M_j} \tag{19}$$

$$\eta_{M_j} = \left(x + yv_{M_jP_{M_j}^n}\right), 0 \le v_{M_jP_{M_j}^n} \le \bar{v}_{M_jP_{M_j}^n} .d_k \tag{20}$$

From equation (19), it is clear that the production transactions units by the production department in the manufacturing firm cannot exceed the actual labor.

Additionally, we assume that the labor availability of the M_j is dependent on the hourly wage w_{M_j}, so that:

$$l_{M_j} = o_{M_j} w_{M_j}, \sum_{j=1}^{J}\sum_{n=1}^{N} l_{M_j} = L \tag{21}$$

where L is the total available labor volume.

To capture cross-department spillovers from labor-efficiency investment, we introduce a distance-decayed spillover index using a grid-distance weighting scheme (22), and map it into a bounded cost-impact factor $e_n^{spillover}$ in (23), which enters the production-cost specification:

$$spillover_n = \varphi\beta\left(\sum_{n=1}^{N} v_{M_jP_{M_j}^n} - v_{M_jP_{M_j}^n}\right) \tag{22}$$

$$e_n^{spillover} = \begin{cases} \sigma \dfrac{spillover_n}{1 + spillover_n}, spillover_n \ne 0. \\ 1, spillover_n = 0 \end{cases} \tag{23}$$

In (22), φ scales the spillover intensity and β governs distance attenuation across production departments. In (23), σ measures the marginal cost impact of the spillover channel.

Furthermore, due to the scale effect of investment, the investment cost for labor efficiency investment in the production department is expressed as $\varpi v_{M_jP_{M_j}^n}^2$, where ϖ is the scale effect coefficient. The salary cost is expressed as $w_{M_j} l_{M_j}$.

On the basis of the above discussion, the profit of M_j, π_{M_j}, can be expressed as:

$$\begin{aligned}\pi_{M_j} = {} & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{M_j} q_{M_jP_{M_j}^n} - (1+\delta s)\sum_{i=1}^{I} \rho_{S_i} q_{S_iM_j} - \sum_{n=1}^{N} e_n^{spillover} f\left(q_{M_jP_{M_j}^n d_k}, v_{M_jP_{M_j}^n}\right) - \sum_{k=1}^{K}\sum_{n=1}^{N} c_{M_jD_k}\left(q_{M_jP_{M_j}^n d_k}\right) - \sum_{k=1}^{K}\sum_{n=1}^{N} c^p_{M_jP_{M_j}^n d_k}\left(q_{M_jP_{M_j}^n d_k}\right) \\ & -\tau\sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP_{M_j}^n d_k} + (\omega - \varepsilon)\left(cap_{M_j} - \alpha_{M_j}\sum_{n=1}^{N}\sum_{k=1}^{K} q_{M_jP_{M_j}^n d_k}\right) - \sum_{n=1}^{N} \varpi v_{M_jP_{M_j}^n}^2 - \sum_{n=1}^{N} w_{M_j} l_{M_j}\end{aligned} \tag{24}$$

In view of Equation (21), we can deduce Equation (24) into Equation (25):

$$\begin{aligned}\pi_{M_j} = {} & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{M_j} q_{M_jP_{M_j}^n d_k} - (1+\delta s)\sum_{i=1}^{I} \rho_{S_i} q_{S_iM_j} - \sum_{n=1}^{N} e_n^{spillover} f\left(q_{M_jP_{M_j}^n d_k}, v_{M_jP_{M_j}^n}\right) - \sum_{k=1}^{K}\sum_{n=1}^{N} c_{M_jd_k}\left(q_{M_jP_{M_j}^n d_k}\right) - \sum_{k=1}^{K}\sum_{n=1}^{N} c^p_{M_jP_{M_j}^n d_k}\left(q_{M_jP_{M_j}^n d_k}\right) \\ & -\tau\sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP_{M_j}^n d_k} + (\omega - \varepsilon)\left(cap_{M_j} - \alpha_{M_j}\sum_{n=1}^{N}\sum_{k=1}^{K} q_{M_jP_{M_j}^n d_k}\right) - \sum_{n=1}^{N} \varpi v_{M_jP_{M_j}^n}^2 - \sum_{n=1}^{N} w_{M_j} l_{M_j} - \sum_{n=1}^{N} o_{M_j} w_{M_j}^2\end{aligned} \tag{25}$$

The profit maximization objective function for manufacturers can be expressed as:

$$\text{Max}\left(\begin{aligned} & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{M_j} q_{M_jP_{M_j}^n d_k} - (1+\delta s)\sum_{i=1}^{I} \rho_{S_i} q_{S_iM_j} - \sum_{n=1}^{N} e_n^{spillover} f\left(q_{M_jP_{M_j}^n d_k}, v_{M_jP_{M_j}^n}\right) - \sum_{k=1}^{K}\sum_{n=1}^{N} c_{M_jd_k}\left(q_{M_jP_{M_j}^n d_k}\right) - \sum_{k=1}^{K}\sum_{n=1}^{N} c^p_{M_jP_{M_j}^n d_k}\left(q_{M_jP_{M_j}^n d_k}\right) \\ & -\tau\sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP_{M_j}^n d_k} + (\omega - \varepsilon)\left(cap_{M_j} - \alpha_{M_j}\sum_{n=1}^{N}\sum_{k=1}^{K} q_{M_jP_{M_j}^n d_k}\right) - \sum_{n=1}^{N} \varpi v_{M_jP_{M_j}^n}^2 - \sum_{n=1}^{N} w_{M_j} l_{M_j} - \sum_{n=1}^{N} o_{M_j} w_{M_j}^2\end{aligned}\right) \tag{26}$$

s.t.

$$q_{S_iM_j} \geq 0, q_{M_jP^n_{M_j}d_k} \geq 0, \rho_{S_i} \geq 0, \rho_{M_j} \geq 0, cap_{M_j} - \alpha_{M_j}\sum_{n=1}^{N}\sum_{k=1}^{K} q_{M_jP^n_{M_j}d_k} \geq 0, o_{M_j} > 0, w_{M_j} > 0,$$
$$q_{M_jP^n_{M_j}d_k} \leq \left(x + yv_{M_jP^n_{M_j}}\right)o_{M_j}w_{M_j}, \bar{v}_{M_jP^n_{M_j}} \geq v_{M_jP^n_{M_j}} \geq 0. \quad (27)$$

and:

$$\sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP^n_{M_j}d_k} \leq \sum_{i=1}^{I} q_{S_iM_j} \quad (28)$$

where Equation (28) simply states that the quantity purchased by consumers from retailers cannot exceed the available inventory.

At this point, Lagrange multipliers λ_{M_j}, γ_{M_j} and ζ_{M_j} are introduced to represent the carbon quota constraint, the transaction flow constraints, and the effective labor capacity constraints, respectively.

On the basis of the above discussion, the Lagrangian function for the manufacturer's profit can be formulated as:

$$\begin{aligned} L_{M_j} = & \sum_{k=1}^{K}\sum_{n=1}^{N} \rho_{M_j} q_{M_jP^n_{M_j}d_k} - (1+\delta s)\sum_{i=1}^{I} \rho_{S_i} q_{S_iM_j} - \sum_{n=1}^{N} e_n^{spillover} f\left(q_{M_jP^n_{M_j}d_k}, v_{M_jP^n_{M_j}}\right) - \sum_{k=1}^{K}\sum_{n=1}^{N} c_{M_jd_k}\left(q_{M_jP^n_{M_j}d_k}\right) - \sum_{k=1}^{K}\sum_{n=1}^{N} c^p_{M_jP^n_{M_j}d_k}\left(q_{M_jP^n_{M_j}d_k}\right) \\ & -\tau\sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP^n_{M_j}d_k} + (\omega-\varepsilon)\left(cap_{M_j} - \alpha_{M_j}\sum_{n=1}^{N}\sum_{k=1}^{K} q_{M_jP^n_{M_j}d_k}\right) - \sum_{n=1}^{N} \varpi v^2_{M_jP^n_{M_j}} - \sum_{n=1}^{N} w_{M_j} l_{M_j} - \sum_{p=1}^{N} o_{M_j} w^2_{M_j} \\ & +\lambda_{M_j}\left(cap_{M_j} - \alpha_{M_j}\sum_{n=1}^{N}\sum_{k=1}^{K} q_{M_jP^n_{M_j}d_k}\right) + \gamma_{M_j}\left(\sum_{i=1}^{I} q_{S_iM_j} - \sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP_nD_k}\right) + \zeta_{M_j}\left(\left(x + yv_{M_jP^n_{M_j}}\right)o_{M_j}w_{M_j} - \sum_{k=1}^{K} q_{M_jP^n_{M_j}d_k}\right) \end{aligned} \quad (29)$$

Owing to the noncooperative game among J competing manufacturers, according to [40], the Nash equilibrium conditions for these manufacturers can be expressed as the following variational inequality:

VIP 2. Determine vector $(\boldsymbol{Q}^{1*}, \boldsymbol{Q}^{2*}, \boldsymbol{\lambda}_M{}^*, \boldsymbol{\gamma}_M{}^*, \boldsymbol{\zeta}_M{}^*, \boldsymbol{v}^*)$ such that

$$\begin{aligned} & \sum_{i=1}^{I}\sum_{j=1}^{J}\left((1+\delta s)\rho_{S_i} - \gamma_{M_j}{}^*\right)\times\left(q_{S_iM_j} - q_{S_iM_j}{}^*\right) \\ & +\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(-\rho_{M_j} + \frac{e_n^{spillover} f\left(q_{M_jP^n_{M_j}d_k}, v_{M_jP^n_{M_j}}\right) + c_{M_jd_k}\left(q_{M_jP^n_{M_j}d_k}\right) + c^p_{M_jP^n_{M_j}d_k}\left(q_{M_jP^n_{M_j}d_k}\right)}{\partial q_{M_jP^n_{M_j}d_k}} + \alpha_{M_j}\left(\omega - \varepsilon + \lambda^*_{M_j}\right) - \gamma_{M_j}{}^* + \zeta_{M_j}{}^* + \tau\right)\times\left(q_{M_jP^n_{M_j}d_k} - q^*_{M_jP^n_{M_j}d_k}\right) \\ & +\sum_{j=1}^{J}\sum_{n=1}^{N}\left(\frac{e_n^{spillover} f\left(q_{M_jP^n_{M_j}d_k}, v_{M_jP^n_{M_j}}\right)}{\partial v_{M_jP^n_{M_j}}} + 2\varpi v_{M_jP^n_{M_j}} - \zeta_{M_j}{}^* o_{M_j} w_{M_j} y\right)\times\left(v_{M_jP^n_{M_j}} - v^*_{M_jP^n_{M_j}}\right) \\ & +\sum_{j=1}^{J}\sum_{n=1}^{N}\left(\sum_{k=1}^{K} q_{M_jP_nD_k} - \left(x + yv_{M_jP_n}\right)o_{M_j}w_{M_j}\right)\times\left(\zeta_{M_j} - \zeta_{M_j}{}^*\right) \\ & +\sum_{j=1}^{J}\left(\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP^n_{M_j}d_k} - cap_{M_j}\right)\times\left(\lambda_{M_j} - \lambda^*_{M_j}\right) \\ & +\sum_{j=1}^{J}\left(\sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP^n_{M_j}d_k} - \sum_{i=1}^{I} q_{S_iM_j}\right)\times\left(\gamma_{M_j} - \gamma_{M_j}{}^*\right) \geq 0 \end{aligned} \quad (30)$$

where $\lambda_M \triangleq \left(\lambda_{M_1}, \lambda_{M_2}, \cdots, \lambda_{M_J}\right)^T$ is the column vector grouped by all Lagrange multipliers for manufacturers' carbon quota constraints; $\gamma_M \triangleq \left(\gamma_{M_1}, \gamma_{M_2}, \cdots, \gamma_{M_J}\right)^T$ is the column vector grouped by all Lagrange multipliers for manufacturers' flow conservation constraints; and $\zeta_M \triangleq \left(\zeta_{M_1}, \zeta_{M_2}, \cdots, \zeta_{M_J}\right)^T$ is the column vector grouped by all Lagrange multipliers for manufacturers' effective labor capacity constraints.

By taking the first-order derivatives of Equation (29) with respect to $\left(q_{S_iM_j}, q_{M_jP_{M_j}^n d_k}, \lambda_{M_j}, \gamma_{M_j}, \zeta_{M_j}, v_{M_jP_{M_j}^n}\right)$, we obtain the following optimality conditions:

$$
\begin{aligned}
&\frac{\partial L_{M_j}}{\partial q_{S_iM_j}} = (1+\delta s)\rho_{S_i} - \gamma_{M_j}{}^{*}, \\
&\frac{\partial L_{M_j}}{\partial q_{M_jP_{M_j}^n d_k}} = -\rho_{M_j} + \frac{e_n^{spillover} f\left(q_{M_jP_{M_j}^n d_k}, v_{M_jP_n}\right) + c_{M_jd_k}\left(q_{M_jP_{M_j}^n d_k}\right) + c_{M_jP_nD_k}^{p}\left(q_{M_jP_{M_j}^n d_k}\right)}{\partial q_{M_jP_{M_j}^n d_k}} + \alpha_{M_j}\left(\omega - \varepsilon + \lambda_{M_j}^{*}\right) - \gamma_{M_j}{}^{*} + \zeta_{M_j}{}^{*} + \tau, \\
&\frac{\partial L_{M_j}}{\partial \zeta_{M_j}} = \sum_{k=1}^{K} q_{M_jP_{M_j}^n} - \left(x + y v_{M_jP_{M_j}^n}\right) o_{M_j} w_{M_j}, \\
&\frac{\partial L_{M_j}}{\partial v_{M_jP_{M_j}^n}} = \frac{e_n^{spillover} f\left(q_{M_jP_{M_j}^n d_k}, v_{M_jP_{M_j}^n}\right)}{\partial v_{M_jP_{M_j}^n}} + 2\varpi v_{M_jP_{M_j}^n} - \zeta_{M_j}{}^{*} o_{M_j} w_{M_j} y, \\
&\frac{\partial L_{M_j}}{\partial \lambda_{M_j}} = \alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP_{M_j}^n d_k} - cap_{M_j}, \\
&\frac{\partial L_{M_j}}{\partial \gamma_{M_j}} = \sum_{k=1}^{K}\sum_{n=1}^{N} q_{M_jP_{M_j}^n d_k} - \sum_{i=1}^{I} q_{S_iM_j}.
\end{aligned}
\tag{31}
$$

Under standard convexity and regularity conditions, the manufacturers' KKT conditions can be equivalent to written in the VI form; hence, solving VI (30) yields the equilibrium solution of the manufacturers' profit-maximization problem [40].

3.5 The Behavior of the Demand Markets and Their Optimality Conditions

At the demand-market tier, consumers choose quantities based on the generalized (delivered) price, defined as the manufacturer's selling price plus the unit transaction cost of accessing the product. Let $c_{d_k} = c_{d_k}\left(q_{M_jP_{M_j}^n d_k}\right)$ denote the demand-market transaction cost and let market demand be price-dependent as $D_k = D_k(\rho_{d_k})$.

To capture in-transit perishability, we introduce a loss coefficient μ, which can be specified as a constant or as flow-dependent to reflect logistics performance; it enters the delivered-price representation by scaling the effective quantity reaching market d_k.

In accordance with [41], we construct a spatial price equilibrium model that considers perishability as:

$$
\begin{cases}
\rho_{M_j} + c_{d_k}\left(q_{M_jP_{M_j}^n d_k}\right) = \mu\rho_{d_k}^{*}, q_{M_jP_{M_j}^n d_k}^{*} > 0 \\
\rho_{M_j} + c_{d_k}\left(q_{M_jP_{M_j}^n d_k}\right) \geq \mu\rho_{d_k}^{*}, q_{M_jP_{M_j}^n d_k}^{*} = 0
\end{cases}
\tag{32}
$$

$$
D_k(\rho_{d_k})
\begin{cases}
= \sum_{j=1}^{J}\sum_{n=1}^{P} \mu q_{M_jP_{M_j}^n d_k}^{*}, \rho_{d_k}^{*} > 0 \\
\leq \sum_{j=1}^{J}\sum_{n=1}^{P} \mu q_{M_jP_{M_j}^n d_k}^{*}, \rho_{d_k}^{*} = 0
\end{cases}
\tag{33}
$$

Equation (32) states that, for each shipment, the delivery unit cost cannot be below the willingness-to-pay (represented by the shadow price), with equality holding whenever a positive flow is shipped.

Equation (33) links the effective inflow after losses to market demand: demand is met when the equilibrium delivered price is positive; otherwise, the complementary slackness implies a zero price.

According to [40], the spatial price equilibrium conditions (32)-(33) for demand markets can be expressed as the following variational inequality:

VIP 3. Determine vector $\left(Q^{2*},\rho_d^*\right)$ such that

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{P}\left(\rho_{M_j}+c_{d_k}\left(q_{M_jP_{M_j}^nd_k}\right)-\mu\rho_{d_k}^*\right)\times\left(q_{M_jP_{M_j}^nd_k}-q_{M_jP_{M_j}^nd_k}^*\right)+\sum_{k=1}^{K}\left(\sum_{j=1}^{J}\sum_{n=1}^{P}\mu q_{M_jP_{M_j}^nd_k}^*-d_k\left(\rho_{d_k}^*\right)\right)\times\left(\rho_{d_k}-\rho_{d_k}^*\right)\geq 0 \tag{34}$$

s.t.

$$q_{M_jP_{M_j}^nd_k}\geq 0,\rho_{d_k}\geq 0.\forall(Q^{2*},\rho_{d_k}^*)\in\Re_+^{JNK+K} \tag{35}$$

where $\rho_d\triangleq\left(\rho_{d_1},\rho_{d_2},\cdots,\rho_{d_K}\right)^T$ is the column vector grouped by all Lagrange multipliers for demand markets.

Economic interpretation of VI (34) represents a complementarity-based market-clearing condition with delivered costs including transaction frictions and perishability. Positive shipments arise only when the delivered cost is consistent with consumers' willingness to pay; otherwise, the corresponding flow is driven to zero.

3.6 The Behavior of the Carbon Trading Center and Its Optimality Conditions

In the carbon trading market, high-emission suppliers purchase allowances to cover compliance deficits, while low-emission manufacturers sell surplus permits. The carbon trading center clears allowance transactions at an exogenous trading price ω and charges an ad-valorem commission ε per transaction. Let c_S^t and c_M^t denote the center's per-transaction operating costs for trades with suppliers and manufacturers, respectively (e.g., matching, verification, and settlement). For tractability, we assume the trading-related transaction frictions are internalized by the center. The center's objective function π_c can thus be expressed as:

$$\pi_c=Max\left(\sum_{i=1}^{I}\left(\varepsilon\left(\alpha_{S_i}\left(1-s\right)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)-c_{S_i}^t\right)+\sum_{j=1}^{J}\left(\varepsilon\left(cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{P}q_{M_jP_{M_j}^nd_k}\right)-c_{M_j}^t\right)\right) \tag{36}$$

s.t.

$$cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{P}q_{M_jP_{M_j}^nd_k}\geq 0,\alpha_{S_i}\left(1-s\right)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\geq 0. \tag{37}$$

$$\sum_{j=1}^{J}\left(cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{P}q_{M_jP_{M_j}^nd_k}\right)\geq\sum_{i=1}^{I}\left(\left(1-s\right)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right) \tag{38}$$

Equation (37) ensures market feasibility (only surplus sellers and deficit buyers participate), while (38) imposes an aggregate balance condition to rule out extreme imbalances and keep allowance trading bounded and economically interpretable.

Considering the carbon quota constraint in Equation (38), a Lagrange multiplier λ_t is introduced. The Lagrangian function L_t can be formulated as:

$$\begin{aligned}L_t=&\sum_{i=1}^{I}\left(\varepsilon\left(\alpha_{S_i}\left(1-s\right)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)-c_{S_i}^t\right)+\sum_{j=1}^{J}\left(\varepsilon\left(cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{P}q_{M_jP_{M_j}^nd_k}\right)-c_{M_j}^t\right)\\&+\lambda_t\left(\sum_{j=1}^{J}\left(cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{P}q_{M_jP_{M_j}^nd_k}\right)-\sum_{i=1}^{I}\left(\left(1-s\right)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)\right)\end{aligned} \tag{39}$$

Then, according to [40], the Nash equilibrium conditions for the carbon trading center can be expressed as the following variational inequality:
VIP 4. Determine vector $\left(\boldsymbol{Q}^{1*},\boldsymbol{Q}^{2*},\lambda_t^*\right)$ such that

$$\begin{aligned}&\sum_{i=1}^{I}\left(\frac{c_{S_i}^t}{\partial q_{S_iM_j}}-\sum_{j=1}^{J}\varepsilon\alpha_{S_i}\left(1-s\right)+\sum_{j=1}^{J}\alpha_{S_i}\lambda_t^*\left(1-s\right)\right)\times\left(q_{S_iM_j}-q_{S_iM_j}^*\right)\\&+\sum_{j=1}^{J}\left(\frac{c_{M_j}^t}{\partial q_{M_jP_{M_j}^nd_k}}+\sum_{k=1}^{K}\sum_{n=1}^{P}\left(\varepsilon\alpha_{M_j}+\alpha_{M_j}\lambda_t^*\right)\right)\times\left(q_{M_jP_{M_j}^nd_k}-q_{M_jP_{M_j}^nd_k}^*\right)\\&+\left(\sum_{j=1}^{J}\left(cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{P}q_{M_jP_{M_j}^nd_k}\right)-\sum_{i=1}^{I}\left(\alpha_{S_i}\left(1-s\right)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)\right)\times\left(\lambda_t-\lambda_t^*\right)\geq 0\end{aligned}\tag{40}$$

By taking the first-order derivatives of Equation (39) with respect to $q_{S_iM_j}$, $q_{M_jP_{M_j}^nd_k}$, and λ_t, we obtain the following optimality conditions:

$$\begin{aligned}&\frac{\partial L_t}{\partial q_{S_iM_j}}=\frac{c_{S_i}^t}{\partial q_{S_iM_j}}-\sum_{j=1}^{J}\varepsilon\alpha_{S_i}\left(1-s\right)+\sum_{j=1}^{J}\alpha_{S_i}\lambda_t^*\left(1-s\right)\\&\frac{\partial L_t}{\partial q_{M_jP_{M_j}^nd_k}}=\frac{c_{M_j}^t}{\partial q_{M_jP_{M_j}^nd_k}}+\sum_{k=1}^{K}\sum_{n=1}^{P}\left(\varepsilon\alpha_{M_j}+\alpha_{M_j}\lambda_t^*\right)\\&\frac{\partial L_t}{\partial\lambda_t}=\sum_{j=1}^{J}\left(cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{P}q_{M_jP_{M_j}^nd_k}\right)-\sum_{i=1}^{I}\left(\alpha_{S_i}\left(1-s\right)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)\end{aligned}\tag{41}$$

Under standard regularity conditions (continuous differentiability and convexity), the trading center's equilibrium can be equivalently characterized by a variational inequality. Hence, the first-order optimality conditions in (42), together with the feasibility constraints in (37)-(38), lead to the VI formulation in VIP-4, which determines the allowance-market equilibrium.

3.7 The Equilibrium Condition of The Supply Chain

By summing the equations representing the optimality conditions for suppliers, manufacturers, demand markets, and the carbon trading market, namely, Equations (10), (30), (34), and (40), the equilibrium model of the entire supply chain network can be obtained. Based on the assumption that the cost functions of all suppliers and manufacturers are continuous and convex, the optimality conditions of the supply chain network can be reduced to the solution of the following variational inequality:
VIP 5. Determine $(\boldsymbol{Q}^{1*},\boldsymbol{Q}^{2*},\boldsymbol{\lambda}_S^{\ *},\boldsymbol{\lambda}_M^{\ *},\boldsymbol{v}^*,\boldsymbol{\gamma}_M^{\ *},\boldsymbol{\zeta}_M^{\ *},\boldsymbol{\rho}_d^*,\lambda_t^*)\in\Re_+^{IJ+JNK+I+J+2JN+1+K}$ such that:

$$
\begin{aligned}
&\sum_{i=1}^{I}\sum_{j=1}^{J}\left(-(1+\delta s)\rho_{S_i} + \frac{f_{S_iM_j}\left(Q^{1*}\right)+c_{S_iM_j}\left(q_{S_iM_j}{}^{*}\right)}{\partial q_{S_iM_j}} + \left((1-s)(\varepsilon+\omega)-\lambda_i{}^{*}\right)\alpha_{S_i}\right)\times\left(q_{S_iM_j}-q_{S_iM_j}{}^{*}\right)\\
&+\sum_{i=1}^{I}\left(cap_{S_i}-\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}{}^{*}\right)\times\left(\lambda_{S_i}-\lambda_{S_i}{}^{*}\right)+\sum_{i=1}^{I}\sum_{j=1}^{J}\left((1+\delta s)\rho_{S_i}{}^{*}-\gamma^{*}\right)\times\left(q_{S_iM_j}-q_{S_iM_j}{}^{*}\right)\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(-\rho_{M_j}+\frac{e_n{}^{spillover}f_{M_jP_{M_j}^nd_k}\left(q_{M_jP_{M_j}^nd_k},v_{M_jP_{M_j}^n}\right)+c_{M_jd_k}\left(q_{M_jP_{M_j}^nd_k}\right)+c^{p}_{M_jP_{M_j}^nd_k}\left(q_{M_jP_{M_j}^nd_k}\right)}{\partial q_{M_jP_{M_j}^nd_k}}+\alpha_{M_j}\left(\omega-\varepsilon+\lambda_j^{*}\right)-\gamma_{M_j}{}^{*}+\zeta_{M_j}{}^{*}+\tau\right)\times\left(q_{M_jP_{M_j}^nd_k}-q^{*}_{M_jP_{M_j}^nd_k}\right)\\
&+\sum_{j=1}^{J}\sum_{n=1}^{N}\left(\frac{e_n{}^{spillover}f\left(q_{M_jP_{M_j}^nd_k},v_{M_jP_{M_j}^n}\right)}{\partial v_{M_jP_{M_j}^n}}+2\varpi v_{M_jP_{M_j}^n}-\zeta_{M_j}{}^{*}o_{M_j}w_{M_j}y\right)\times\left(v_{M_jP_{M_j}^n}-v^{*}_{M_jP_{M_j}^n}\right)\\
&+\sum_{j=1}^{J}\sum_{n=1}^{N}\left(\sum_{k=1}^{K}q_{M_jP_{M_j}^nd_k}-\left(x+yv_{M_jP_n}\right)o_{M_j}w_{M_j}\right)\times\left(\zeta_{M_j}-\zeta_{M_j}{}^{*}\right)\\
&+\sum_{j=1}^{J}\left(\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{N}q_{M_jP_{M_j}^nd_k}-cap_{M_j}\right)\times\left(\lambda_{M_j}-\lambda^{*}_{M_j}\right)\\
&+\sum_{j=1}^{J}\left(\sum_{k=1}^{K}\sum_{n=1}^{N}q_{M_jP_{M_j}^nd_k}-\sum_{i=1}^{I}q_{S_iM_j}\right)\times\left(\gamma_{M_j}-\gamma_{M_j}{}^{*}\right)\\
&\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\rho_{M_j}+c_{d_k}\left(q_{M_jP_{M_j}^nd_k}\right)-\mu\rho^{*}_{d_k}\right)\times\left(q_{M_jP_{M_j}^nd_k}-q^{*}_{M_jP_{M_j}^nd_k}\right)+\sum_{k=1}^{K}\left(\sum_{j=1}^{J}\sum_{n=1}^{N}\mu q^{*}_{M_jP_{M_j}^nd_k}-D_k\left(\rho^{*}_{d_k}\right)\right)\times\left(\rho_{d_k}-\rho^{*}_{d_k}\right)\\
&\sum_{i=1}^{I}\left(\frac{c^{t}_{S_i}}{\partial q_{S_iM_j}}-\sum_{j=1}^{J}\varepsilon\alpha_{S_i}(1-s)+\sum_{j=1}^{J}\alpha_{S_i}\lambda_t^{*}(1-s)\right)\times\left(q_{S_iM_j}-q^{*}_{S_iM_j}\right)+\sum_{j=1}^{J}\left(\frac{c^{t}_{M_j}}{\partial q_{M_jP_{M_j}^nd_k}}+\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\varepsilon\alpha_{M_j}+\alpha_{M_j}\lambda_t^{*}\right)\right)\times\left(q_{M_jP_nD_k}-q^{*}_{M_jP_{M_j}^nd_k}\right)\\
&+\left(\sum_{j=1}^{J}\left(cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{N}q_{M_jP_{M_j}^nd_k}\right)-\sum_{i=1}^{I}\left(\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)\right)\times\left(\lambda_t-\lambda_t^{*}\right)\geq 0
\end{aligned}
\tag{43}
$$

For convenience in subsequent use, the variational inequality in Equation (42) can be rewritten in the following form:

VIP 6. Determine $X^{*}\in\mathcal{K}$ such that:

$$
\left\langle \boldsymbol{F}\left(\boldsymbol{X}^{*}\right),\boldsymbol{X}-\boldsymbol{X}^{*}\right\rangle\geq 0,\quad \forall\boldsymbol{X}\in\mathcal{K} \tag{42}
$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product in the Euclidean space $(IJ+JNK+I+J+2JN+1+K)$.

The solution $X=\left(\boldsymbol{Q}^{1*},\boldsymbol{Q}^{2*},\boldsymbol{\lambda}_S{}^{*},\boldsymbol{\lambda}_M{}^{*},v^{*}_{M_jP_{M_j}^n},\boldsymbol{\gamma}_M{}^{*},\boldsymbol{\zeta}_M{}^{*},\rho^{*}_{d_k},\boldsymbol{\lambda}_t^{*}\right)$ represents the equilibrium solution of the supply chain network.

$$
\mathcal{K}\equiv\left\{\left(\boldsymbol{Q}^{1*},\boldsymbol{Q}^{2*},\boldsymbol{\lambda}_S{}^{*},\boldsymbol{\lambda}_M{}^{*},v^{*}_{M_jP_n},\boldsymbol{\gamma}_M{}^{*},\boldsymbol{\zeta}_M{}^{*},\rho^{*}_{d_k},\boldsymbol{\lambda}_t^{*}\right)\middle|\left(\boldsymbol{Q}^{1*},\boldsymbol{Q}^{2*},\boldsymbol{\lambda}_S{}^{*},\boldsymbol{\lambda}_M{}^{*},v^{*}_{M_jP_n},\boldsymbol{\gamma}_M{}^{*},\boldsymbol{\zeta}_M{}^{*},\rho^{*}_{d_k},\boldsymbol{\lambda}_t^{*}\right)\in R_{+}^{IJ+JNK+I+J+2JN+1+K}\right\}
$$

$$
\boldsymbol{F}(\boldsymbol{X})\equiv\left(F^{\boldsymbol{Q}^1}_{S_iM_j},F^{\boldsymbol{Q}^2}_{M_jP_{M_j}^nd_k},F^{\boldsymbol{\lambda}_S}_{S_i},F^{\boldsymbol{\lambda}_M}_{M_j},F^{v_{M_jP_{M_j}^n}}_{M_jP_n},F^{\boldsymbol{\gamma}_M}_{M_j},F^{\boldsymbol{\zeta}_M}_{M_jP_n},F^{\rho_{d_k}}_{d_k},F_t^{\lambda t}\right)_{i=1,\dots,I;\,j=1,\dots,J;\,k=1,\dots,K;\,n=1,\dots N;\,t=1}
$$

where

$$
\begin{aligned}
F_{S_iM_j}^{\boldsymbol{Q}^1} &= \sum_{j=1}^{J}\left(\frac{f_{S_iM_j}\left(Q^{1*}\right)+c_{S_iM_j}\left(q_{S_iM_j}{}^{*}\right)}{\partial q_{S_iM_j}} + \left((1-s)(\varepsilon+\omega)-\lambda_{S_i}{}^{*}\right)\alpha_{S_i}-\gamma_{M_j}{}^{*}\right) \\
&\quad +\left(\frac{c_{S_i}^{t}}{\partial q_{S_iM_j}}-\sum_{j=1}^{J}\varepsilon\alpha_{S_i}(1-s)+\sum_{j=1}^{J}\alpha_{S_i}\lambda_t^{*}(1-s)\right) \\
F_{M_jP_{M_j}^nd_k}^{\boldsymbol{Q}^2} &= \sum_{j=1}^{K}\sum_{n=1}^{N}\left(\frac{e_n^{\,spillover} f\left(q_{M_jP_{M_j}^nd_k}, v_{M_jP_{M_j}^n}\right)+c_{M_jd_k}\left(q_{M_jP_{M_j}^nd_k}\right)+c^{p}_{M_jP_{M_j}^nd_k}\left(q_{M_jP_{M_j}^nd_k}\right)}{\partial q_{M_jP_{M_j}^nd_k}} + \alpha_{M_j}\left(\omega-\varepsilon+\lambda_j^{*}\right)-\gamma_{M_j}{}^{*}+\zeta_{M_j}{}^{*}+\tau+c_{d_k}\left(q_{M_jP_{M_j}^nd_k}\right)-\mu\rho_{d_k}^{*}\right) \\
&\quad +\left(\frac{c_{S_i}^{t}}{\partial q_{M_jP_nD_k}}+\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\varepsilon\alpha_{M_j}+\alpha_{M_j}\lambda_t^{*}\right)\right) \\
F_{S_i}^{\lambda_{S_i}} &= cap_{S_i}-\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}{}^{*} \\
F_{M_j}^{\lambda_{M_j}} &= \alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{N}q_{M_jP_{M_j}^nd_k}-cap_{M_j} \\
F_{M_jP_{M_j}^n}^{v_{M_jP_{M_j}^n}} &= \sum_{n=1}^{N}\left(\frac{e_n^{\,spillover} f\left(q_{M_jP_{M_j}^nd_k}, v_{M_jP_{M_j}^n}\right)}{\partial v_{M_jP_{M_j}^n}}+2\varpi v_{M_jP_{M_j}^n}-\zeta_{M_j}{}^{*}o_{M_j}w_{M_j}y\right) \\
F_{M_jP_n}^{\zeta_{M_j}} &= \sum_{k=1}^{K}q_{M_jP_{M_j}^nd_k}-\left(x+yv_{M_jP_{M_j}^n}\right)o_{M_j}w_{M_j} \\
F_{M_j}^{\gamma_{M_j}} &= \sum_{k=1}^{K}\sum_{n=1}^{N}q_{M_jP_{M_j}^nd_k}-\sum_{i=1}^{I}q_{S_iM_j} \\
F_k^{\rho_{d_k}} &= \sum_{j=1}^{J}\sum_{n=1}^{N}\mu q^{*}_{M_jP_{M_j}^nd_k}-d_k\left(\rho_{D_k}^{*}\right) \\
F_t^{\lambda_t} &= \sum_{j=1}^{J}\left(cap_{M_j}-\alpha_{M_j}\sum_{k=1}^{K}\sum_{n=1}^{P}q_{M_jP_{M_j}^nd_k}\right)-\sum_{i=1}^{I}\left(\alpha_{S_i}(1-s)\sum_{j=1}^{J}q_{S_iM_j}-cap_{S_i}\right)
\end{aligned}
$$

4. Qualitative Properties

This section establishes qualitative properties of the solution to VI (43), with emphasis on existence and uniqueness. Because the feasible set of VI (43) is not necessarily compact, existence does not follow directly from standard continuity/convexity arguments. Following the classical VI existence framework, we introduce a bounded closed convex subset $\mathcal{K}_b$ and derive sufficient conditions under which VI (43) admits a solution.

We define a bounded, closed, and convex set $\mathcal{K}_b$ as follows:

$$
\mathcal{K}_b=\left\{\left(\boldsymbol{Q}^1,\boldsymbol{Q}^2,\lambda_{S_i},\lambda_{M_j},v_{M_jP_{M_j}^n},\gamma_{M_j},\zeta_{M_j},\rho_{d_k},\lambda_t\right)\middle|\begin{array}{c}0\le\boldsymbol{Q}^1\le b_1;0\le\boldsymbol{Q}^2\le b_2;0\le\lambda_i\le b_3;0\le\lambda_j\le b_4;\\ 0\le v_{M_jP_{M_j}^n}\le b_5;0\le\gamma_{M_j}\le b_6;0\le\zeta_{M_j}\le b_7;0\le\rho_{d_k}\le b_8;0\le\lambda_t\le b_9\end{array}\right\}
$$

where the vector b is set as $b\equiv(b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9)\ge 0$. And $\boldsymbol{Q}^1\le b_1,\boldsymbol{Q}^2\le b_2,\lambda_{S_i}\le b_3,\lambda_{M_j}\le b_4,v_{M_jP_{M_j}^n}\le b_5,\gamma_{M_j}\le b_6,\zeta_{M_j}\le b_7,\rho_{d_k}\le b_8,\lambda_t\le b_9$. This means that for all i, j, k, n, we have $q_{S_iM_j}\le b_1,q_{M_jP_{M_j}^nd_k}\le b_2,\lambda_{S_i}\le b_3,\lambda_{M_j}\le b_4,v_{M_jP_{M_j}^n}\le b_5,\gamma_{M_j}\le b_6,\zeta_{M_j}\le b_7,\rho_{d_k}\le b_8,\lambda_t\le b_9$, ensuring that $\mathcal{K}_b$ is a compact subset of $R_+^{IJ+JNK+I+J+2JN+1+K}$.

Researcher has demonstrated standard variational inequality theory that since $\mathcal{K}_b$ is a compact set and the function $\boldsymbol{F}$ is a continuously differentiable and convex function [42], the following VI (44) has at least one solution $\boldsymbol{X}^b\in\mathcal{K}_b$:

$$
\left\langle \boldsymbol{F}\left(\boldsymbol{X}^b\right),\boldsymbol{X}-\boldsymbol{X}^b\right\rangle\ge 0,\ \ \forall \boldsymbol{X}^b\in\mathcal{K}_b \tag{43}
$$

On this basis, we can derive the following theorem:

Lemma 1. VI (44) has a solution in the set $\mathcal{K}_b$ if and only if there exists a $b > 0$, and the VI (43) has a solution under the following condition:

$$\boldsymbol{Q}^{1b} \le b_1, \boldsymbol{Q}^{2b} \le b_2, \lambda_{S_i}^b \le b_3, \lambda_{M_j}{}^b \le b_4, v^b{}_{M_j P_{M_j}^n} \le b_5, \gamma_{M_j}{}^b \le b_6, \zeta_{M_j}{}^b \le b_7, \rho_{d_k}^b \le b_8, \lambda_t^b \le b_9 \quad (44)$$

Lemma 1 links the existence of a solution to VI (43) to the existence of a solution within the bounded set $\mathcal{K}_b$ In particular, under the boundedness requirement in (45), one can select sufficiently large upper bounds b such that the associated bounded VI admits a solution, which in turn ensures the existence of a solution to VI (43).

Theorem 1. (Existence). Suppose that there exist positive constants M, L, and R with R > 0 such that:

$$\frac{\partial f_{S_i M_J}\left(\boldsymbol{Q}^1\right)}{\partial q_{S_i M_j}} + \frac{\partial c_{S_i M_J}\left(q_{S_i M_J}\right)}{\partial q_{S_i M_j}} + \frac{\partial c_{S_i}^t}{\partial q_{S_i M_j}} \ge M, \forall\ q_{S_i M_J} \ge L, \forall i, j, n\ .$$
$$\frac{\partial e_n^{spillover} f\left(q_{M_j P_{M_j}^n d_k}, v_{M_j P_{M_j}^n}\right) + \partial c_{M_j D_k}\left(q_{M_j P_{M_j}^n d_k}\right) + \partial c^p_{M_j P_{M_j}^n d_k}\left(q_{M_j P_{M_j}^n d_k}\right)}{\partial q_{M_j P_{M_j}^n d_k}} + \frac{\partial c_{M_j}^t}{\partial q_{M_j P_{M_j}^n d_k}} \ge M, \forall\ q_{M_j P_{M_j}^n d_k} \ge L, \forall j, n, k\ . \quad (45)$$

$$D_k(\rho_{d_k}) \le L, \forall \rho_{d_k} > R, \forall k. \quad (46)$$

Proof: The proof is based on Lemma 1 and follows the existence proof provided by Nagurney [43].

Conditions (47)-(48) rule out degenerate cases with vanishing marginal costs under unbounded flows and ensure that demand remains bounded at sufficiently high delivered prices, which together guarantee the existence of an equilibrium.

Theorem 2 (Monotonicity). If the relevant cost functions are continuously differentiable and convex in decision variables, the demand-side transaction cost is nondecreasing in the generalized price, and the demand function is nonincreasing in the generalized price, then the VI mapping $\boldsymbol{F}(\cdot)$ in VI (43) is monotone on $\mathcal{K}$, i.e., (49) holds. That is:

$$\left\langle \boldsymbol{F}\left(\boldsymbol{X}^{'}\right) - \boldsymbol{F}\left(\boldsymbol{X}^{"}\right), \boldsymbol{X}^{'} - \boldsymbol{X}^{"}\right\rangle \ge 0, \quad \forall \boldsymbol{X}^{'}, \boldsymbol{X}^{"} \in \mathcal{K}. \quad (47)$$

where $\boldsymbol{F}\left(\boldsymbol{X}^{'}\right)$ represents the gradient of the cost functions and other relevant functions participating in the variational inequality.

Monotonicity guarantees the well-posedness of the VI and serves as a key sufficient condition for existence/uniqueness under additional assumptions.

Proof: Let $\boldsymbol{X}^{'} = (\boldsymbol{Q}^{1'}, \boldsymbol{Q}^{2'}, \lambda_{S_i}{}^{'}, \lambda_{M_j}{}^{'}, v^{'}_{M_j P_{M_j}^n}, \gamma_{M_j}{}^{'}, \zeta_{M_j}{}^{'}, \rho_{d_k}^{'}, \lambda_t^{'})$, $\boldsymbol{X}^{"} = (\boldsymbol{Q}^{1"}, \boldsymbol{Q}^{2"}, \lambda_{S_i}{}^{"}, \lambda_{M_j}{}^{"}, v^{"}_{M_j P_{M_j}^n}, \gamma_{M_j}{}^{"}, \zeta_{M_j}{}^{"}, \rho_{dk}^{"}, \lambda_t^{"})$, $\boldsymbol{X}^{'} \in \mathcal{K}$, $\boldsymbol{X}^{"} \in \mathcal{K}$; then,

$$
\begin{aligned}
\left\langle \boldsymbol{F}\left(\boldsymbol{X}^{'}\right)-\boldsymbol{F}\left(\boldsymbol{X}^{"}\right),\boldsymbol{X}^{'}-\boldsymbol{X}^{"}\right\rangle =& \sum_{i=1}^{I}\sum_{j=1}^{J}\left(\frac{\partial f_{S_iM_j}\left(\boldsymbol{Q}^{1'}\right)}{\partial q_{S_iM_j}}-\frac{\partial f_{S_iM_j}\left(\boldsymbol{Q}^{1"}\right)}{\partial q_{S_iM_j}}\right)\times\left(q_{S_iM_j}{}^{'}-q_{S_iM_j}{}^{"}\right)\\
&+\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\frac{\partial c_{S_iM_j}\left(q_{S_iM_j}{}^{'}\right)}{\partial q_{S_iM_j}}-\frac{\partial c_{S_iM_j}\left(q_{S_iM_j}{}^{"}\right)}{\partial q_{S_iM_j}}\right)\times\left(q_{S_iM_j}{}^{'}-q_{S_iM_j}{}^{"}\right)\\
&+\sum_{i=1}^{I}\left(\frac{\partial c_{S_i}^{t'}}{\partial q_{S_iM_j}}-\frac{\partial c_{S_i}^{t"}}{\partial q_{S_iM_j}}\right)\times\left(q_{S_iM_j}-q_{S_iM_j}{}^{"}\right)\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\frac{\partial e_n^{spillover}f_{M_jP_{M_j}^nd_k}\left(\boldsymbol{Q}^{2'},v^{'}_{M_jP_{M_j}^n}\right)}{\partial q_{jk}}-\frac{\partial e_n^{spillover}f_{M_jP_{M_j}^nd_k}\left(\boldsymbol{Q}^{2"},v^{"}_{M_jP_{M_j}^n}\right)}{\partial q_{jk}}\right)\times\left(q_{M_jP_{M_j}^nd_k}{}^{'}-q_{M_jP_{M_j}^nd_k}{}^{"}\right)\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\frac{\partial c_{M_jd_k}\left(q_{M_jP_{M_j}^nd_k}{}^{'}\right)}{\partial q_{M_jP_{M_j}^nd_k}}-\frac{\partial c_{M_jd_k}\left(q_{M_jP_{M_j}^nd_k}{}^{"}\right)}{\partial q_{M_jP_{M_j}^nd_k}}\right)\times\left(q_{M_jP_{M_j}^nd_k}{}^{'}-q_{M_jP_{M_j}^nd_k}{}^{"}\right)\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(\frac{\partial c^{p}_{M_jP_{M_j}^nd_k}\left(q_{M_jP_{M_j}^nd_k}{}^{'}\right)}{\partial q_{M_jP_{M_j}^nd_k}}-\frac{\partial c^{p}_{M_jP_{M_j}^nd_k}\left(q_{M_jP_{M_j}^nd_k}{}^{"}\right)}{\partial q_{M_jP_{M_j}^nd_k}}\right)\times\left(q_{M_jP_{M_j}^nd_k}{}^{'}-q_{M_jP_{M_j}^nd_k}{}^{"}\right)\\
&+\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(c_{d_k}\left(q_{M_jP_{M_j}^nd_k}{}^{'}\right)-c_{d_k}\left(q_{M_jP_{M_j}^nd_k}{}^{"}\right)\right)\times\left(q_{M_jP_{M_j}^nd_k}{}^{'}-q_{M_jP_{M_j}^nd_k}{}^{"}\right)\\
&+\sum_{j=1}^{J}\left(\frac{\partial c_{M_j}^{t'}}{\partial q_{M_jP_{M_j}^nd_k}}-\frac{\partial c_{M_j}^{t"}}{\partial q_{M_jP_{M_j}^nd_k}}\right)\times\left(q_{M_jP_{M_j}^nd_k}{}^{'}-q_{M_jP_{M_j}^nd_k}{}^{"}\right)\\
&+\sum_{j=1}^{J}\sum_{n=1}^{N}\left(\frac{\partial e_n^{spillover}f_{M_jP_{M_j}^nd_k}\left(Q^{2'},v^{'}_{M_jP_{M_j}^n}\right)}{\partial v_{M_jP_{M_j}^n}}-\frac{\partial e_n^{spillover}f_{M_jP_{M_j}^nd_k}\left(Q^{2"},v^{"}_{M_jP_{M_j}^n}\right)}{\partial v_{M_jP_{M_j}^n}}\right)\times\left(v_{M_jP_{M_j}^n}{}^{'}-v_{M_jP_{M_j}^n}{}^{"}\right)\\
&+\sum_{k=1}^{K}\left(D_k\left(\rho_{d_k}{}^{"}\right)-D_k\left(\rho_{d_k}{}^{'}\right)\right)\times\left(\rho^{'}_{d_k}-\rho^{"}_{d_k}\right)\\
&=(I)+(II)+(III)+(IV)+(V)+(VI)+(VII)+(VIII)+(IVV).
\end{aligned}
\tag{48}
$$

By convexity and monotonicity of the involved functions, each inner-product term is nonnegative.

Furthermore, since the transaction cost function c_{d_k} in the demand market is monotonically increasing and the demand function D_k is monotonically decreasing with respect to the generalized price, we have:

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{n=1}^{N}\left(c_{d_k}\left(q_{M_jP_{M_j}^nd_k}{}^{'}\right)-c_{d_k}\left(q_{M_jP_{M_j}^nd_k}{}^{"}\right)\right)\times\left(q_{M_jP_{M_j}^nd_k}{}^{'}-q_{M_jP_{M_j}^nd_k}{}^{"}\right)\geq 0 \tag{49}$$

$$\sum_{k=1}^{K}\left(D_k\left(\rho_{d_k}{}^{"}\right)-D_k\left(\rho_{d_k}{}^{'}\right)\right)\times\left(\rho^{'}_{d_k}-\rho^{"}_{d_k}\right)\geq 0 \tag{50}$$

Combining the above results, we conclude the following:

$$\left\langle \boldsymbol{F}\left(\boldsymbol{X}^{'}\right)-\boldsymbol{F}\left(\boldsymbol{X}^{"}\right),\boldsymbol{X}^{'}-\boldsymbol{X}^{"}\right\rangle \geq 0,\ \ \forall \boldsymbol{X}^{'},\boldsymbol{X}^{"}\in\mathcal{K}. \tag{51}$$

Thus, Variational Inequality (43) is monotonic.

Theorem 3 (Strict monotonicity). We assume that all the conditions of Theorem 2 are satisfied and further assume that at least one of the cost functions is strictly monotonic. Under this assumption, the function in variational inequality becomes strictly monotonic. Specifically:

$$\left\langle \boldsymbol{F}\left(\boldsymbol{X}^{'}\right)-\boldsymbol{F}\left(\boldsymbol{X}^{"}\right),\boldsymbol{X}^{'}-\boldsymbol{X}^{"}\right\rangle \geq 0,\ \ \forall \boldsymbol{X}^{'},\boldsymbol{X}^{"}\in\mathcal{K}. \tag{52}$$

This strict monotonicity ensures that the solution to variational inequality (43) is unique under the given conditions.

Theorem 4 (Uniqueness). Under the conditions of Theorem 2, there exists a unique solution $\left(\boldsymbol{Q}^{1*},\boldsymbol{Q}^{2*},\lambda_{S_i}{}^{*},\lambda_{M_j}{}^{*},v^{*}_{M_jP^n_{M_j}},\gamma^{*}{}_{M_j},\zeta^{*}{}_{M_j},\rho^{*}_{d_k},\lambda^{*}_t\right)$ that satisfies the equilibrium conditions of the supply chain. In other words, if variational inequality (46) has a solution, then the solution is unique and must be $\left(\boldsymbol{Q}^{1*},\boldsymbol{Q}^{2*},\lambda_{S_i}{}^{*},\lambda_{M_j}{}^{*},v^{*}_{M_jP^n_{M_j}},\gamma^{*}{}_{M_j},\zeta^{*}{}_{M_j},\rho^{*}_{d_k},\lambda^{*}_t\right)$.

Theorem 5 (Lipschitz continuity). The function in variational inequality (42) satisfies Lipschitz continuity, specifically:

$$\|\boldsymbol{F}\left(\boldsymbol{X}^{'}\right)-\boldsymbol{F}\left(\boldsymbol{X}^{"}\right)\|\leq L\|\boldsymbol{X}^{'}-\boldsymbol{X}^{"}\|,\forall\boldsymbol{X}^{'},\boldsymbol{X}^{"}\in\mathcal{K},L>0 \quad (53)$$

where L > 0 and L is the Lipschitz constant. To ensure this property, the following conditions must be satisfied:

For all cost functions $f_{S_iM_ji},c_{S_iM_j},e_n{}^{spillover}f_{M_jP^n_{M_j}d_k},c_{M_jd_k},c^p_{M_jP^n_{M_j}d_k},c_{d_k},c^t_{S_i},c^t_{M_j}$, the functions have bounded second-order derivatives. The transaction cost function and the demand function in the demand market have bounded first-order derivatives.

5. The Algorithm

To solve the problem of variational inequality, researchers use different techniques such as improved projection techniques, quasi-Newton techniques, and projection dynamical systems. But since the problem has similarities with other research works that the current study draws from, the improved projection algorithm is therefore utilized to solve the problem [43]. The standard form of the algorithm to solve the variational inequality problem (43) has four steps that include initialization, iterative calculation, refinement, and convergence check [44].

6. Case Study and Numerical Examples

6.1 Case on China-EU Garlics

This section reports numerical experiments based on the China-EU garlic trade setting. The case is used as an illustrative institutional and market background to design economically meaningful scenarios (e.g., export frictions, sustainability-related compliance pressure, and an ETS-pilot environment), rather than to claim statistical representativeness or to fit parameters for forecasting.

Garlic is a high-volume agri-food export product with well-known cross-border frictions and strong quality requirements in destination markets. These features make it suitable for examining how (i) trade frictions and (ii) carbon-regulatory constraints jointly shape firms' production, transactions, and investment decisions in an oligopolistic supply-chain network.

As an illustrative background, we use the China-EU garlic trade, where China is the world's major producer and exporter (about 2 million metric tons in 2022) and the EU imports around 200,000 metric tons annually. The EU is largely supplied by China (e.g., accounting for 25.5% of extra-EU garlic imports in 2022), while market access is shaped by quota/tariff arrangements (e.g., an annual quota of about 48,225 metric tons) [45]. In addition, the destination market has increasingly emphasized sustainability and carbon-footprint disclosure requirements, which raises compliance pressure and motivates low-carbon technology investment for continued market entry [46]. These institutional features jointly justify our modeling of policy-induced trade frictions and carbon-compliance incentives within the proposed multi-tier network equilibrium framework.

On the destination-market side, evolving EU sustainability and climate policies increasingly require stricter product standards and, in some cases, carbon-footprint disclosure for agri-food imports. On the domestic side, China's ETS pilots can provide a relevant institutional reference for modeling allowance constraints and trading. In our framework, these policy environments motivate the inclusion of a carbon-trading center and the corresponding compliance trading mechanisms. The ETS-pilot component provides the carbon constraint and trading-price channel, labor-efficiency investment represents the operational adjustment channel, and proximity-based spillovers govern how such adjustments propagate across divisions; the three numerical examples are structured to separately reveal these channels and their interaction under an oligopolistic equilibrium.

The next examples illustrate the equilibrium solution of an oligopolistic international agricultural supply chain network model with dual investments, i.e., carbon quota trading, under ETS pilots through numerical examples. These examples are solved via the modified projection method described in Section 5 and computed via MATLAB on Windows. The examples are initialized as follows: the transaction volumes between suppliers and manufacturers $q_{S_iM_j}$ and between manufacturers and demand markets $q_{M_jP^n_{M_j}d_k}$ are initialized to 1.00. The low-carbon technology investment levels s and labor efficiency investments $v_{M_jP^n_{M_j}}$ are initially set to 0.00. The Lagrange multipliers associated with the carbon quota and labor capacity constraints are also initialized to 0.00. The convergence tolerance is set to 10^{-6}; that is, the algorithm is considered to have converged when the absolute value of the difference between each variable at two successive iterations differs by no more than this value. The parameter ζ in the algorithm is set to 0.1.

Parameter setting and validation strategy. Our paper is a mechanism-oriented equilibrium simulation rather than a forecasting exercise. The China–EU garlic trade is used to motivate the institutional background and scenario design (e.g., ETS-pilot compliance and target-market sustainability requirements), rather than to calibrate parameters for trend fitting. The functional forms adopted in this section follow standard specifications widely used in the agri-food supply chain modeling literature [47-50]. Credibility is evaluated through systematic sensitivity and robustness checks, key inputs are varied over reasonable ranges, and equilibrium outcomes are examined for consistent comparative-statics patterns (reported in Tables 2-5).

6.2 Baseline Example with No Cross-Border Trade and a Single Oligopolistic Supplier

Figure 2 depicts the supply-chain network topology for Example 1. Example 1 serves as a stylized baseline with one supplier and two competing manufacturers serving a single demand market (no cross-border trade). Although it is a baseline setting, we retain both low-carbon technology investment (supplier side) and labor-efficiency investment (manufacturer side) to isolate their equilibrium effects under quota-based carbon regulation.

Functional forms for costs and demand follow standard specifications widely used in the network-equilibrium and agri-food supply-chain modeling literature [47-50], with minor adaptations to reflect (i) investment-dependent production efficiency and (ii) the carbon-trading mechanism embedded in our framework. The specific design is as follows:

The production cost function for the supplier is as follows:

$$f_{S_1M_1} = 0.8q_{S_1M_1}{}^2 + 3.5q_{S_1M_1}, f_{S_1M_2} = 0.8q_{S_1M_2}{}^2 + 3.5q_{S_1M_2}$$

The transaction cost function between the supplier and the manufacturer is as follows:

$$c_{S_1M_1} = 0.8q_{S_1M_1}{}^2 + 0.5q_{S_1M_1}, c_{S_1M_2} = 0.8q_{S_1M_2}{}^2 + 0.5q_{S_1M_2}$$

The production cost function for the manufacturer is as follows:

$$f_{M_1P^1_{M_1}d_1} = 0.5q_{M_1P^1_{M_1}d_1}{}^2\left(0.3 + 0.7e^{\frac{-3v_{P^1_{M_1}d_1}}{1+v_{P^1_{M_1}d_1}}}\right), f_{M_2P^1_{M_2}d_1} = 0.5q_{M_2P^1_{M_2}d_1}{}^2\left(0.3 + 0.7e^{\frac{-3v_{P^1_{M_2}d_1}}{1+v_{P^1_{M_2}d_1}}}\right)$$

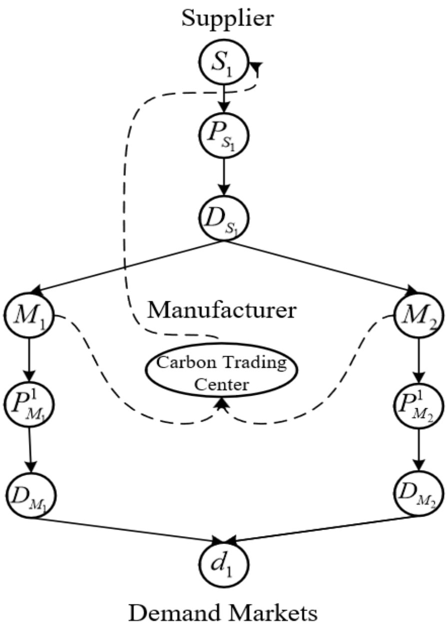

Fig. 2. Supply chain network topology for Example 1

The transaction cost function between the manufacturer and the demand market is as follows:

$$c_{M_1 d_1} = 0.1 q_{M_1 P^1_{M_1} d_1}{}^2 + 0.5 q_{M_1 P^1_{M_1} d_1}, c_{M_2 d_1} = 0.1 q_{M_2 P^1_{M_2} d_1}{}^2 + 0.5 q_{M_2 P^1_{M_2} d_1}$$

The transportation cost function for the manufacturer is as follows:

$$c^p_{M_1 P^1_{M_1} d_1} = 0.1 q_{M_1 P^1_{M_1} d_1}{}^2 + 0.1 q_{M_1 P^1_{M_1} d_1}, c^p_{M_2 P_1 D_1} = 0.1 q_{M_2 P^1_{M_2} d_1}{}^2 + 0.1 q_{M_2 P^1_{M_2} d_1}$$

The demand function and transaction cost function in the demand market are as follows:

$$D_{d_1} = -4 P_{d_1} + 1000, \quad c_{d_1} = 0.05 \left(q_{M_1 P^1_{M_1} d_1} + q_{M_2 P^1_{M_2} d_1} \right)^2$$

The transaction costs for the carbon trading center with the supplier and manufacturer are as follows:

$$c^t_{S_1} = 0.1 \left((1-s) a_{S_1} \left(q_{S_1 M_1} + q_{S_1 M_2} \right) - cap_{S_1} \right)^2, c^t_{M_j} = 0.1 \left(cap_{M_1} - a_{S_1} \left(q_{M_1 P^1_{M_1} d_1} + q_{M_2 P^1_{M_2} d_1} \right) \right)^2$$

Carbon-emission parameters are set as follows. The supplier receives an emission quota of 3 units with an emission coefficient of 0.25, while the manufacturer receives an emission quota of 3 units with an emission coefficient of 0.05. The carbon trading price is exogenously fixed at 0.5 per unit. Supplier and manufacturer emissions are denoted by E_{S_i} and E_{M_j}, respectively.

We use this stylized baseline to isolate the equilibrium roles of (i) low-carbon investment on the supply side and (ii) labor-efficiency determinants on the manufacturing side, before introducing richer cross-border settings.

Table 2 presents the sensitivity analysis results for low-carbon investment levels s (range 0 - 0.5). As s increases, transaction volumes between suppliers and manufacturers grow by approximately 15%, rising from 17.01% to 19.58%. Transaction volumes between manufacturers and demand markets increase by about 27%, climbing from 14.89% to 18.89%. Correspondingly, the supplier transaction price surged from 11.43 to 60.95, while the market price also climbed significantly from 85.87 to 164.73.

Meanwhile, profits for both parties increased significantly: supplier profits rose from 145.67 to 2479.01, while manufacturer profits increased from 1300.14 to 1822.04. Carbon emissions also decreased markedly: supplier emissions fell from 4.25% to 2.45%, a 42% reduction; manufacturer emissions increased from 0.74% to 0.94%. Notably, as the s-value increased, the manufacturer's labor efficiency investment level decreased from 12.37 to 11.03, indicating a strategic shift in investment focus as improved upstream carbon performance drives overall supply chain efficiency growth.

Table 2
Sensitivity analysis of the low-carbon investment level

Low-Carbon Investment Levels	$q_{S_iM_j}{}^*$	$q_{M_jP^n_{M_j}d_k}{}^*$	$v_{M_jP^n_{M_j}}{}^*$	$\rho_{S_i}{}^*$	$\rho_{M_j}{}^*$	$\rho^*_{d_k}$	$\pi^*_{S_i}$	$\pi^*_{M_j}$	π^*	E_{S_i}	E_{M_j}
0	17.01	14.89	12.37	11.43	85.87	244.76	145.67	1300.14	2745.95	4.2525	0.7445
0.1	17.61	15.72	11.72	8.86	102.52	244.48	768.11	1385.32	3538.71	3.9623	0.7861
0.2	18.09	16.57	11.67	21.33	117.55	244.17	1113.11	1484.91	4083.01	3.6180	0.8285
0.3	18.61	17.31	11.49	34.24	132.94	243.93	1508.01	1580.33	4668.63	3.2551	0.8650
0.4	19.09	18.06	11.14	47.52	148.78	243.66	1965.22	1690.31	5345.72	2.6852	0.9032
0.5	19.58	18.89	11.03	60.95	164.73	243.36	2479.01	1822.04	5123.21	2.4475	0.9445

Mechanism interpretation. The sensitivity pattern in Table 2 is economically uneven across outcomes. Changes in traded quantities and prices are material as s increases, indicating that supplier-side decarbonization relaxes the effective carbon pressure embedded in the quota-and-trading mechanism and allows a higher-throughput equilibrium. Meanwhile, the manufacturer's labor-efficiency investment exhibits a limited decline over the same range, which is best interpreted as a reallocation response: when upstream carbon performance improves, the marginal benefit of labor-efficiency spending becomes weaker relative to expanding shipments under the looser effective carbon constraint. Importantly, the emissions response is asymmetric across tiers, upstream emissions fall markedly, while downstream emissions rise only slightly, suggesting that upstream abatement does not automatically translate into lower downstream emissions when equilibrium throughput expands.

Overall, Table 2 provides a baseline illustration that supplier-side low-carbon investment can generate a higher-activity equilibrium with higher prices and profits in this simplified setting, while the accompanying emissions reduction is concentrated upstream. At the same time, the downstream emissions increase is small in magnitude and should not be over-interpreted as a deterioration of environmental performance; rather, it reflects the throughput-expansion channel in the equilibrium adjustment. These results are therefore used to motivate the subsequent cross-border scenarios, where trade frictions and policy stringency may amplify, dampen, or even overturn these baseline comparative-statics.

Table 3
Sensitivity analysis of the labor efficiency determinant

Labor Efficiency Determinanty	$q_{S_iM_j}{}^*$	$q_{M_jP^n_{M_j}d_k}{}^*$	$v_{M_jP^n_{M_j}}{}^*$	$\rho_{S_i}{}^*$	$\rho_{M_j}{}^*$	$\rho^*_{d_k}$	$\pi^*_{S_i}$	$\pi^*_{M_j}$	π^*	E_{S_i}	E_{M_j}
0	17.01	14.89	0	11.43	85.87	244.76	145.67	1300.14	2745.95	4.2525	0.7445
0.1	17.61	15.72	11.72	8.86	102.52	244.48	768.11	1385.32	3538.71	4.4025	0.7863
0.125	17.61	15.74	11.94	9.11	102.83	244.47	776.91	1387.71	3552.34	4.4025	0.7871
0.15	17.63	15.75	12.39	9.52	103.32	244.47	786.52	1388.65	3563.82	4.4075	0.7875
0.175	17.65	15.81	12.96	10.21	104.18	244.45	806.13	1389.72	3585.41	4.4125	0.7877
0.2	17.71	15.87	14.01	11.54	105.87	244.43	840.92	1403.81	3648.53	4.4275	0.7935

Table 3 reports the sensitivity of equilibrium outcomes to the labor-efficiency determinant γ (from 0 to 0.2). Across this range, both upstream and downstream transaction volumes rise only modestly roughly within single-digit percentages, indicating that changes in y generate limited quantitative adjustments in the baseline network.

Prices respond asymmetrically: the supplier-side transaction price decreases slightly at low y and then stabilizes, whereas the manufacturer-side price increases. Profits increase for both tiers, but the magnitude is noticeably smaller than the response observed under suppliers' low-carbon investment

Environmental outcomes show no clear improvement in this baseline setting: total emissions for both suppliers and manufacturers increase slightly as y rises, suggesting that efficiency-driven throughput expansion can offset potential per-unit efficiency gains.

Mechanism interpretation. Labor-efficiency investment affects the equilibrium mainly through the manufacturers' operational cost and throughput decisions, rather than directly relaxing the carbon quota constraint. Therefore, compared with suppliers' low-carbon investment in Table 2, the equilibrium adjustments in Table 3 are more moderate. The increase in emissions alongside higher labor efficiency can be interpreted as a rebound effect: efficiency improvements support higher market activity and throughput, which may raise total emissions even if per-unit operations become more efficient. This difference in channels helps connect the numerical findings to the theoretical design in H2.

The sensitivity analysis in Table 3 indicates that the equilibrium responses to labor-efficiency investment are more moderate than those under supplier low-carbon investment (Table 2). At equilibrium, both parties' profits increase, but the environmental improvement remains limited, suggesting that labor-efficiency investment alone does not guarantee emission reductions in this simplified setting. This benchmark result provides a reference for the cross-border scenarios in Examples 2 and 3.

6.3 Example with Cross-Border Trade with Labor Efficiency Investment in a Single Manufacturing Department

Building Example 1, Example 2 extends the baseline network to a cross-border setting with two manufacturers and two demand markets (domestic and foreign), while keeping the same functional forms and cost parameters. The purpose is to examine how international trade frictions and department-level labor-efficiency investment reshape equilibrium outcomes in a multi-division structure. The supply chain network structure for Example 2 is illustrated in Figure 3.

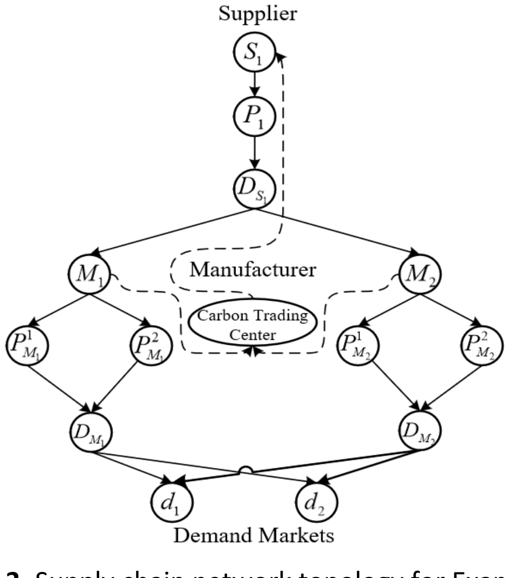

Fig. 3. Supply chain network topology for Example 2

Each manufacturer operates two production divisions, one serving the domestic market and the other serving the foreign market, which allows us to model market-specific decisions under different trade conditions.

In this example, only Manufacturer M_1's foreign-oriented division undertakes labor-efficiency investment. This targeted design isolates the export-channel incentive, i.e., when delivered costs embed trade and compliance frictions, efficiency investment becomes a direct lever for competitiveness, without introducing additional investment decisions that would blur interpretation.

We further incorporate a distance-based spillover term to capture how efficiency improvements in one division may affect other divisions' operating costs. After varying y(2) (the labor-efficiency determinant of M_1's foreign division), the sensitivity results are reported in Table 4.

Table 4
Sensitivity analysis of the labor efficiency determinant

Labor Efficiency Determinant y(2)	$q_{M_1 P^1_{M_1} d_1}{}^*$	$q_{M_1 P^2_{M_1} d_2}{}^*$	$q_{M_2 P^1_{M_2} d_1}{}^*$	$q_{M_2 P^2_{M_2} d_2}{}^*$	$v_{M_1 P^2_{M_1}}{}^*$	$\rho_{S_1 M_1}{}^*$	$\rho_{S_1 M_2}{}^*$	$\rho_{P^1_{M_1}}{}^*$	$\rho_{P^2_{M_1}}{}^*$	$\rho_{P^1_{M_2}}{}^*$	$\rho_{P^2_{M_2}}{}^*$	E_{M_1}	E_{M_2}
0	23.3043	29.3586	23.3043	29.3586	0	21.4559	21.4559	21.9559	26.0691	21.9559	26.0691	0.5466	0.5466
0.5	25.3854	29.6453	25.3571	29.3985	11.1149	18.8178	20.3891	20.3194	19.3178	20.8891	25.1322	0.5503	0.5476
0.75	25.3853	29.6453	25.3572	29.3985	11.1155	18.8179	20.3893	20.3193	19.3179	20.8893	25.1322	0.5503	0.5476
1	25.3854	29.6451	25.3572	29.3982	11.1163	18.8173	20.3893	20.3193	19.3173	20.8893	25.1323	0.5503	0.5476
1.25	25.3849	29.6452	25.3569	29.3985	11.1175	18.8177	20.3892	20.3191	19.3177	20.8892	25.1321	0.5503	0.5476
1.5	25.3853	29.6453	25.3572	29.3984	11.1194	18.8179	20.3891	20.3189	19.3179	20.8891	25.1310	0.5503	0.5476
1.75	25.4661	0	25.4390	29.4683	50	18.1208	18.7377	18.6208	2431.9817	19.2311	23.4937	0.2547	0.5491

Labor Efficiency Determinant y(2)	$\rho^*_{P^1_{M_1} d_1}$	$\rho^*_{P^2_{M_1} d_2}$	$\rho^*_{P^1_{M_2} d_1}$	$\rho^*_{P^2_{M_2} d_2}$	$\pi^*_{M_1}$	$\pi^*_{M_2}$	$\pi^*_{S_1}$	π^*	$Spillover_{P^1_{M_1}}$	$Spillover_{P^1_{M_2}}$	$Spillover_{P^2_{M_2}}$	$q_{S_1 M_1}{}^*$	$q_{S_1 M_2}{}^*$
0	195.5716	259.8161	195.5716	259.8161	970.7547	970.7547	1129.92	3071.43	0	0	0	52.6629	52.6629
0.5	195.5574	259.7489	195.5623	259.8066	857.6287	945.5497	1075.98	2879.16	0.5002	0.2779	0.2223	55.0306	54.7556
0.75	195.5574	259.7490	195.5623	259.8067	857.6252	945.5509	1075.99	2879.17	0.5002	0.2779	0.2223	55.0306	54.7557
1	195.5574	259.7489	195.5622	259.8066	857.6265	954.5461	1075.97	2888.14	0.5002	0.2779	0.2223	55.0305	54.7553
1.25	195.5573	259.7490	195.5623	259.8067	857.6221	954.5469	1075.97	2888.14	0.5003	0.2779	0.2223	55.0301	54.7553
1.5	195.5574	259.7491	195.5623	259.8066	857.6191	945.5398	1075.98	2879.14	0.5004	0.2708	0.2224	55.0306	54.7556
1.75	195.5432	266.6663	195.5481	259.7904	337.5836	903.6855	744.97	1986.23	2.2512	1.2501	1.0021	25.4661	54.9073

Table 4 reports the equilibrium responses to changes in the labor-efficiency determinant y(2), which governs labor-efficiency investment in Manufacturer M_1's international division. As y(2) increases from 0 to 1.5, equilibrium quantities across divisions rise modestly and then enter a plateau region approximately when y(2)∈[0.5,1.5]. When y(2) reaches 1.75, the equilibrium shifts to a boundary outcome: the output of M_1's international division collapses to zero, indicating a structural reconfiguration driven by internal substitution under intensified efficiency incentives.

Pricing outcomes mirror the quantity pattern. For moderate y(2), prices adjust downward initially and remain relatively stable within the plateau interval, suggesting that efficiency improvements are largely absorbed through cost and allocation adjustments rather than triggering aggressive price competition. At y(2)=1.75, the associated price variable exhibits an extreme jump, which should be interpreted as a shadow-value signal under binding constraints rather than a regular market-clearing price, consistent with the division-exit outcome.

In profitability and emissions, the responses are asymmetric across firms. As y(2) rises within the moderate range, total profits adjust gradually, while emissions for M_1 remain broadly stable. Once the equilibrium switches at y(2)=1.75, M_1's activity contraction leads to a discrete drop in its emissions, whereas M_2's emissions remain largely unaffected. Therefore, the environmental improvement observed at the threshold is mainly driven by structural exit rather than incremental efficiency gains.

Investment and spillover indicators show diminishing returns. While the investing division maintains moderate investment levels over the plateau interval, the marginal spillover benefit weakens as y(2) increases. At the threshold, the investing division's implied cost can surge, reflecting inefficient over-investment incentives that ultimately destabilize the division's participation in equilibrium.

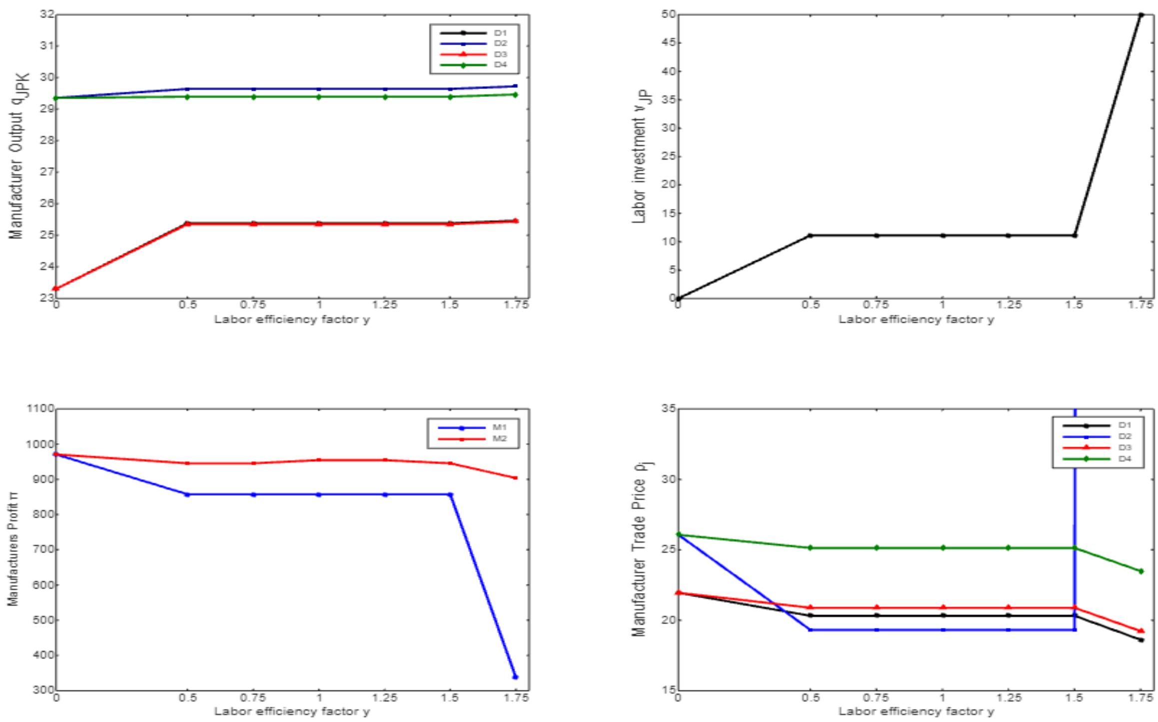

Fig. 4. Sensitivity analysis of the labor efficiency determinant

These complex dynamics are clearly visualized in Figure 4. This figure illustrates how the gradual increase of y(2) initially brings about marginal improvements in output and cost efficiency, but may eventually disrupt the equilibrium structure of the supply chain. This case highlights the nonlinear effects of unilateral efficiency investment in multi-layer networks under carbon constraints, and sometimes even counterintuitive effects.

Compared with the benchmark case, the cross-border, multi-division structure makes the equilibrium more sensitive to localized efficiency shocks. The results indicate that efficiency investment concentrated in a single export-oriented division may initially benefit the network, but excessive incentives can induce internal substitution and reallocation that ultimately eliminates that division's equilibrium output.

Mechanism interpretation. The pattern in Table 4 supports a localized-efficiency with proximity spillovers channel: for moderate y(2), spillovers are positive but bounded, yielding plateau-like responses; beyond a threshold, diminishing returns and internal substitution push the equilibrium toward a boundary solution where one division's optimal output becomes zero. This pattern is consistent with the role of spillovers and diminishing returns embedded in our model design. The evidence is consistent with H3 and complements the baseline patterns documented in Example 1.

Overall, Example 2 suggests that labor-efficiency investment in export-oriented divisions should be calibrated rather than maximized: moderate investment yields stable improvements, while overly strong efficiency incentives may induce structural reconfiguration and division exit. This example is intended as an illustrative stress-test of the model's nonlinear equilibrium mechanism, and it motivates the subsequent scenarios that examine robustness under alternative cross-border configurations.

6.4 Example with Cross-Border Trade with Labor Efficiency Investment in All Manufacturing Departments

Example 3 generalizes Example 2 by allowing labor-efficiency investment in all manufacturing divisions rather than only one. Figure 5 depicts the corresponding cross-border network. We keep the network topology and baseline cost specifications unchanged, and vary only the labor-efficiency determinant to isolate how system-wide efficiency investment reshapes equilibrium outcomes under the same carbon-trading setting.

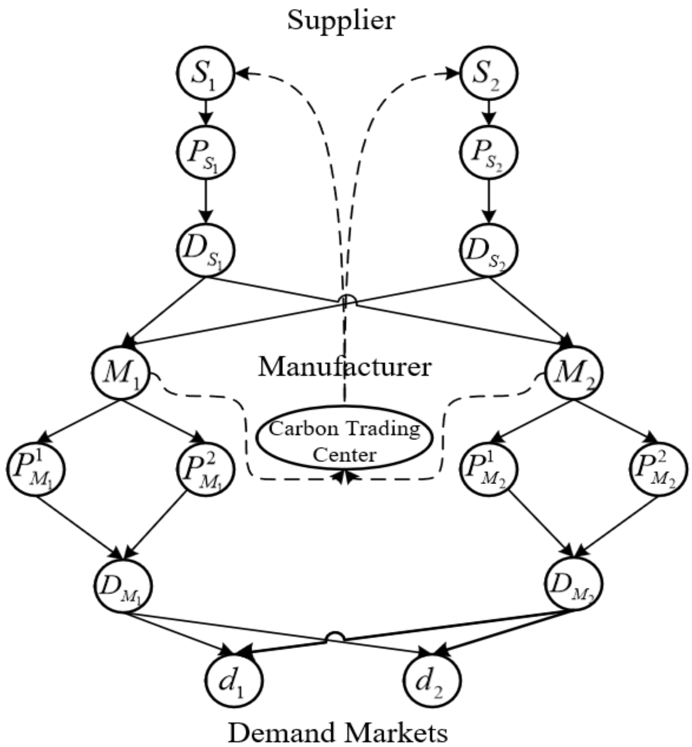

Fig. 5. Supply chain network topology for Example 3

In this setting, labor-efficiency investment is available in all four manufacturing divisions. We use y(2) to parametrize the efficiency potential of M_1's international division, so that changing y(2) represents an asymmetric improvement within an otherwise system-wide investment regime.

This example is intended as an illustrative robustness check on whether the nonlinearity observed in Example 2 persists when efficiency investment is not localized.

Table 5
Sensitivity analysis of the labor efficiency determinant

Labor Efficiency Determinant	$q_{M_1P^1_{M_1}d_1}{}^*$	$q_{M_1P^2_{M_1}d_2}{}^*$	$q_{M_2P^1_{M_2}d_1}{}^*$	$q_{M_2P^2_{M_2}d_2}{}^*$	$v_{M_1P^1_{M_1}}{}^*$	$v_{M_1P^2_{M_1}}{}^*$	$v_{M_2P^1_{M_2}}{}^*$	$v_{M_2P^2_{M_2}}{}^*$	$\rho_{P^1_{M_1}}{}^*$	$\rho_{P^2_{M_1}}{}^*$	$\rho_{P^1_{M_2}}{}^*$	$\rho_{P^2_{M_2}}{}^*$	$\rho^*_{P^1_{M_1}d_1}$
0	23.3043	29.3586	23.3043	29.3586	0	0	0	0	21.9559	26.0691	21.9559	26.0691	195.5716
0.5	25.6843	29.6987	25.6843	29.6988	10.2332	10.2373	10.2332	10.2131	14.4813	18.2304	14.4815	18.2303	195.5049
0.75	25.6843	29.6987	25.6843	29.6986	10.2047	10.2396	10.2047	10.1746	14.4821	18.2341	14.4829	18.2315	195.5052
1	25.6842	29.6985	25.6842	29.6986	10.1515	10.2439	10.1515	10.1109	14.4837	18.2341	14.4852	18.2337	195.5051
1.25	25.6841	29.6985	25.6840	29.6985	10.0661	10.2503	10.0660	10.0512	14.4861	18.2381	14.4889	18.2372	195.5052
1.5	25.6839	29.6981	25.6837	29.6983	9.9347	10.2599	9.9347	9.8737	14.4899	18.2438	14.4944	18.2425	195.5051
1.75	25.6922	0	25.6815	25.6922	3.1866	50	3.1866	3.1154	14.2929	1258.7553	14.5463	18.2218	195.5037

Labor Efficiency Determinant	$\rho^*_{P^2_{M_1}d_2}$	$\rho^*_{P^1_{M_2}d_1}$	$\rho^*_{P^2_{M_2}d_2}$	$\pi^*_{M_1}$	$\pi^*_{M_2}$	$Spillover_{P^1_{M_1}}$	$Spillover_{P^2_{M_1}}$	$Spillover_{P^1_{M_2}}$	$Spillover_{P^2_{M_2}}$	E_{S_i}	E_{M_j}	$q_{S_1}{}^*$	$q_{S_2}{}^*$
0	259.8161	195.5716	259.8161	970.7547	970.7547	0	0	0	0	0.5466	0.5466	52.6629	52.6629
0.5	259.7366	195.5050	259.7367	764.9520	764.9622	0.9208	0.9205	0.9202	0.9211	0.5538	0.5538	55.3829	55.3831
0.75	259.7367	195.5049	259.7366	764.9891	765.0058	0.9194	0.9177	0.9178	0.9193	0.5538	0.5538	55.3829	55.3829
1	259.7366	195.5050	259.7367	765.0481	765.0909	0.9171	0.9126	0.9137	0.9159	0.5538	0.5538	55.3827	55.3828
1.25	259.7368	195.5050	259.7366	765.1521	765.2206	0.9132	0.9047	0.9073	0.9105	0.5538	0.5538	55.3826	55.3825
1.5	259.7368	195.5051	259.7367	765.2991	765.4166	0.9075	0.8926	0.8979	0.9023	0.5537	0.5537	55.3821	55.3820
1.75	266.6663	195.5054	259.7365	282.3009	765.5215	2.3920	0.2850	1.2199	1.4571	0.2569	0.5538	25.6922	51.3737

Table 5 presents the sensitivity analysis results of Example 3, where the labor efficiency investment is implemented simultaneously in all production departments, and the investment coefficient y(2) is different. Initially, when y(2) increased from 0 to 0.5, the trading volumes of all four sectors rose. However, as y(2) continued to increase from 0.5 to 1.75, the trading volume of each department gradually declined, indicating that excessive efficiency potential might lead to saturation and strategic production adjustments. The equilibrium exhibits a strong response over a low-to-moderate range of y(2) , where traded quantities and prices move in the same direction. Beyond this range, the response becomes weak or non-monotonic, indicating diminishing returns and reallocation across divisions rather than proportional scaling.

Regarding labor efficiency investments, as y(2) increases from 0.5 to 1.75, the labor investment costs across all four departments show a consistent decreasing trend. This downward investment trend indicates that when the efficiency potential of one department increases, all departments will strategically reduce their labor investment, which suggests an optimized scenario that recognizes diminishing returns. Furthermore, contrary to the pattern observed in Example 2, the spillover effect between departments shows a downward trend, indicating that when efficiency improvements occur simultaneously throughout the entire supply chain, the interaction dynamics are also different.

These observed trends are clearly depicted in Figure 6, which visualizes these responses and distinguishes the smooth-adjustment region at moderate y from the regime-switch region at high y(2). While moderate efficiency y(2) potential supports coordinated throughput and profit adjustments, extremely high triggers y(2) corner outcomes, signaling structural reconfiguration.

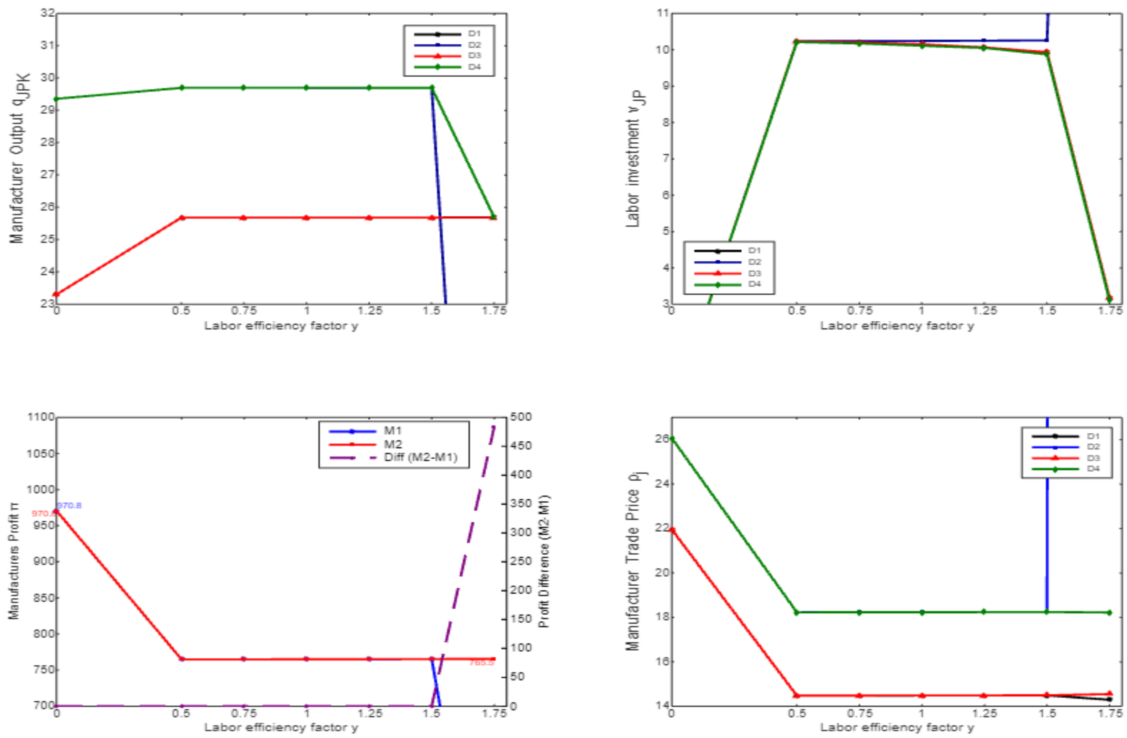

Fig. 6. Sensitivity analysis of the labor efficiency determinant.

Compared with Example 2, Example 3 yields a more coordinated profit response at moderate y(2) levels. However, once y(2) becomes sufficiently high, the equilibrium may switch to a corner regime, some shipments collapse and the associated shadow prices jump, indicating structural reconfiguration rather than a smooth market adjustment.

Mechanism interpretation. Compared with Example 2, allowing investment across all divisions shifts the adjustment channel from a localized spillover-driven response to a network-wide reallocation response. Moderate efficiency improvements expand feasible throughput, but higher y(2) amplifies diminishing returns and re-optimization across divisions, which can compress spillovers and trigger corner-type equilibria. This pattern is consistent with H3 (joint role of efficiency investment and proximity-mediated effects) and, under cross-border compliance pressure, aligns with H2 regarding stronger operational adjustments in export-oriented segments.

Overall, Example 3 suggests that system-wide labor-efficiency investment can improve cross-border performance within a moderate range of efficiency potential, but extreme values may induce reallocation and corner outcomes. These results are illustrative of mechanism-based equilibrium shifts in the proposed network model, rather than a claim of empirical representativeness.

7. Summary and Conclusions

7.1 Summary of The Study

This paper develops a variational-inequality (VI) equilibrium framework for an oligopolistic international agri-food supply chain operating under an ETS-pilot quota constraint and allowance trading. The model endogenizes production and transaction decisions across upstream suppliers, downstream manufacturers, demand markets, and a carbon-trading center, while incorporating two investment channels that are economically salient under climate regulation: supplier-side low-carbon technology improvement and manufacturer-side labor-efficiency investment with proximity-mediated spillovers. A China-EU garlic trade context is used as an illustrative setting to motivate the institutional background and scenario design, and numerical examples are reported to examine comparative-statics patterns under progressively richer network structures.

7.2 Main Findings and Economic Interpretation

The simulations indicate that tightening carbon-related constraints and enabling allowance trading do not only affect compliance costs; they reallocate quantities and prices across tiers through network interactions. Upstream low-carbon improvement reduces the supplier's emissions but can simultaneously expand downstream activity, so system-wide emissions may not decline uniformly across tiers. This cross-tier reallocation pattern corresponds to the upstream investment channel emphasized in H1, and it cautions against evaluating policy effectiveness using tier-specific indicators alone.

Labor-efficiency investment affects equilibrium primarily through cost and throughput adjustments and the spillover structure across nearby divisions. For moderate efficiency levels, equilibrium responses are smooth and gains are broadly shared; when efficiency potential becomes sufficiently high in a localized division, the equilibrium may shift toward a corner-type outcome, accompanied by discontinuities in some quantities and shadow-value-like price movements. This nonlinearity is consistent with H3. In cross-border settings, stricter target-market sustainability requirements can amplify firms' incentives to adjust investment and operational choices, reinforcing the compliance-pressure channel highlighted in H2.

7.3 Implications

For firms, the results suggest that more investment is not always optimal in an interconnected network. Balanced investment strategies that account for diminishing returns, internal substitution, and spillover attenuation are more likely to yield stable gains than highly concentrated unilateral improvements.

For policymakers, the findings imply that ETS design and complementary instruments should be evaluated at the supply-chain level: allowance trading and quota stringency can reshape market activity and emissions distribution across tiers. In cross-border contexts, coordinating ETS incentives with credible disclosure/verification and with mechanisms that facilitate coordinated adjustment along supply-chain flows can improve policy coherence and reduce unintended emission reallocation.

7.4 Limitations and Future Work

The numerical experiments are mechanism-oriented and use an illustrative trade context to motivate scenarios rather than to provide calibrated forecasts. Functional forms and parameters are chosen for tractability and comparative-statics clarity, so magnitudes should be interpreted cautiously. Future work may strengthen external validity through calibration and empirical evaluation, extend the framework to richer logistics/perishability structures, and quantify welfare and distributional effects under alternative ETS and disclosure policy packages.

In conclusion, the proposed VI-based equilibrium framework provides a mechanism-oriented tool for examining how quota constraints, carbon trading, and dual investment decisions jointly shape economic and environmental outcomes in international agri-food supply chains.

Author Contributions

Conceptualization, T.W.; methodology, T.W.; software, T.W.; validation, T.W., and H.H.; formal analysis, T.W.; investigation, T.W.; resources, T.W.; data curation, T.W.; writing—original draft preparation, T.W.; writing—review and editing, H.H.; visualization, H.H.; supervision, H.H.; project

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All relevant data is contained within the article. The original contributions presented in the study are included in the article and supplementary material, further inquiries can be directed to the corresponding author.

Conflicts of Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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