

A Fundamental Cause of Economic Crisis—A Macro-economic Game between the Real Economic Sector and Monetary Sector

Koji Akimoto

ABSTRACT

This study examines a fundamental cause of the economic crisis from which the current capitalist economy is suffering. We posit that the cause is an imbalance between the real economic sector and the monetary sector. To test this hypothesis, we construct a differential game. Players are the agents selected by the two sectors. We adopt linear objective functions, because if we adopt concave objective functions, the analysis is restricted to examining the first and the second order conditions to maximize the Hamiltonians. A main theoretical originality of this study is to describe how players and policy makers should play to ensure balance between the real economic and the monetary sectors in the capitalist economy. The auxiliary equations do not have any stable steady state points under the linear functions. The game requires two rules to have a solution. One rule is the *strategy rule* that requires players to adopt a strategy that makes the strategy exchange point coincide with the steady state point in reaction to the other player's strategy. The other rule is the *policy rule* that requires the policy-maker (Central Bank) to control the strategy exchange point of the monetary sector by undertaking buying operation. It is shown that the distribution rate of GDP to the monetary sector has a rigorous range for the game to have a solution. In addition, the solution has the character of a *natural economy*. We conclude that large deviations from the solution of the game (which we consider as deviations from the *natural economy*) are a fundamental cause of economic crisis. We investigate actual transitions of the distribution rate in USA, Euro area and Japan to prove our hypothesis in identifying the fundamental cause.

Keywords: economic crisis, unbalance of the real economic sector and the monetary sector, strategy rule, policy rule, natural economy, differential game

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Introduction

A remarkable trend in the capitalist economy since the 1980s has been the rapid expansion of the world's financial assets and corresponding rapid increases of GDP in areas of the world with high assets¹. Indeed, this trend is the result of the current Keynesian policy. These substantial financial assets require interest rates which have two sources. One is the money created by credit creation and the other is the value produced by the real economic sector. However, if the interest rates are paid from credit creation, this action then falls within the monetary sector. Therefore, the problem to be analyzed is the latter case. This case should be considered as the event where the value produced by the real economic sector is absorbed into the monetary sector. The resulting expansion of money causes a rapid increase in absorption of value from the real economic sector.

The expansion of money should be considered as being destructive to the economy's balance. From the viewpoint of economic theory, this destruction is considered as a deviation from macro-economic equilibrium and therefore deviation from a *natural economy*. Indeed, a *natural economy* is not a synonym for macro-economic equilibrium. However, as we consider below, a *natural economy*,—despite differences in definition between various viewpoints, involves an economic equilibrium. Therefore, deviation from such an equilibrium is considered as a deviation from a *natural economy*.

As a result, it is more meaningful to examine the *natural economy*. As the economist Adam Smith (1776) described, the market price fluctuates around *the natural price*. This fluctuation has two important implications. One is that a *natural economy* constitutes the basic structure of the economy. Therefore, to study the *natural economy* means to analyse the basic law that manages and controls the economy. The other is that any large deviation from it entails high risk. Such risks therefore provide an important alarm for controlling

¹ For instance, see <http://www.meti.go.jp/report/tsuhaku2008/.../html/i1120000.html>

the economy without rigorous rules. We cannot artificially control the economy by ignoring the *natural economy*.

When we study the *natural economy* by Adam Smith. (1776), we should also analyze *natural interest rates*. After Smith, many economists tried to define a *natural interest rate* from macro-economic viewpoint. First, we should highlight the definition by Wicksell (1898). Wicksell defined a *natural interest rate* as being neutral for the price level of a real market. More precisely, it is the rate at which demand is equal to supply in the real market making a capital market unnecessary. As is well known, this definition had an impact on Keynes. In his book, *A Treatise on Money* (1930), Keynes constructed

the fundamental equation whereby a *natural interest rate* was defined as a rate that made investment equal to savings. At *the natural interest rate*, the price level is equal to the monetary income per output paid for the production factor. However, a situation wherein investment equals savings implies dependences on the shapes of both investment and saving functions. For instance, if an innovation is expected to occur, the expectation of this innovation will change the investment function and the *natural interest rate* will increase. In this case, no information is given with respect to developments in the production process. However, it is absolutely certain that the *natural interest rate* depends upon the structure of the production process.

Pasinetti(1981) proposed a different approach to that of Wicksell. Pasinetti concentrated on the structures of the production process and innovation. His approach is based on the theory of labour. First, he began with the condition of full employment of labour and capital stock, using a *vertically integrated analysis* defined by the multi-sector model. Second, he introduced the concept of *natural rate of profit* and constructed a *natural economy*. Finally, he introduced financial assets to the *natural economy* and proved the emergence of a rate of interest. After these preparations, Pasinetti defined the own-rate of interest for each commodity and rigorously analysed the relation between the *nominal rate of interest* and the *real rates of interest (the standard real rate of interest)*. Defining the concept of interest rates at each stage, he finally reached the definition of *the natural rate of interest*.

Natural interest rates defined by Wicksell and Pasinetti are the representative ones and both these definitions are based on macro-economic equilibrium. However, the

macro-economy has manifold structures and therefore has multiple concepts of equilibrium. As will be analysed in Section 3, the solution of the game in this study is also a macro-economic equilibrium and has the character of the *natural economy*. Therefore, a large deviation from equilibrium implies the deviation from the *natural economy*.

Let us return to the actual economy. A remarkable character we should indicate is the rapid expansion of the monetary sector. If we investigate financial policies in USA, Euro area and Japan, we observe that the common feature of these areas is the rapid increase in monetary bases. Corresponding to this increase, financial assets in the world increased and reached 3.5 times as the world's GDP in 2006². This imbalance implies a large deviation from the *natural economy*.

Finally, we discuss the theoretical originality of our game-model. The pioneer in this field is Lancaster(1971). Lancaster uses a differential game to indicate the inefficiency of capitalism in his study. Players in his model are capitalists and workers. This study has been expanded by Basar and Olsder(1982), Basar,Haurie and Ricci(1985), Pohjola(1985), Kaitala and Pohjola(1990), Benabou, Roland and Tirole(2006) and so on. These studies analyse the existence of an equilibrium along an infinite horizon and the inefficiency of capitalism. Players are also capitalists and workers. However, the game in this study differs from these studies in two respects. The first is a difference of economic viewpoint. The players in the games in the above studies are also capitalists and workers. Workers receive all of the wealth at the beginning and then give their savings to capitalists. Therefore, these studies do not incorporate the distribution process in their models. In contrast, we focus on the basic structure of the current economy. That is, the economy is constructed using the real economic sector and the monetary sector. Players are the agents selected by the two sectors. We introduce an endogenously determined distribution process. This model describes the absorption process by the monetary sector and enables us to analyse how much the monetary sector can receive for the economy to be sustained. The second difference is important. Lancaste(1971) constructed his model by adopting linear objective functions under a finite horizon. Indeed, his descendants revised Lancaster's model by introducing concave objective functions under an infinite horizon. However, by adopting these

² www.meti.go.jp/report/tsuhaku2008/.../html/i1120000.html

conditions, their analyses are restricted to examining the first and the second order conditions to maximize the Hamiltonians. Therefore, we adopt linear objective functions under an infinite horizon. These linear functions conversely enable us to describe how players should play in the capitalist economy. However, the auxiliary equations do not have any stable steady state points. This implies the capitalist economy is unstable by itself and needs some rules to be sustained. Specially, the game requires two rules to have a solution. One is the *strategy rule* that requires players to adopt the strategy that makes the strategy exchange point coincide with the steady state point, reacting the other player's strategy. The other is the *policy rule* that requires the policy-maker (Central Bank) to control the strategy exchange point of the monetary sector by engaging in buying operations. The distribution rate for the monetary sector has a rigorous range for the game to have its solution. In addition, the solution has the character of the *natural economy*. We conclude that a large deviation from the solution of the game (which we consider as a deviation from the *natural economy*) is a fundamental cause of economic crisis.

We construct the macro-economic game in Section 2 and analyse it in Section 3. We have adopted a differential game. However, the game has no stable steady point. We demonstrate how that the aid of financial policy is inevitable for the game to have its solution. Here, we propose the rule of financial policy that controls the rate by which the monetary sector absorbs the value produced by the real economic sector.

Finally, in Section 4, we apply our game-model to the actual economy. The area we select includes USA, Euro area and Japan. Moreover, we prove that the actual economy largely deviates from the game equilibrium, hence from the *natural economy*.

2. Model

Let us consider the economy comprising the real economic sector that produces GDP and the monetary sector that holds financial assets and invests part of them in the real economic sector. The produced GDP is distributed between the real economic and the monetary sector. We assume that each sector has an agent who makes the decision for consumption and investment in the production in occurring the real economic sector.

We denote the production function of the macro-economy by

$$Y = F(K, L) = K^\theta L^{1-\theta}, \quad (0 < \theta < 1) \quad (1)$$

where Y , K and L represent GDP, capital stock and labor input, respectively. θ is a constant. We also denote the price of capital stock and the wage rate by p_k and w , respectively. They are constants. Then, we obtain

$$L = \frac{1-\theta}{\theta} \cdot \frac{p_k}{w} K \quad (2)$$

by the optimal condition for the input of factors; $\frac{F_K}{p_k} = \frac{F_L}{w}$ where F_K and F_L represent the partial derivatives of F with respect to K and L , respectively. Therefore, we get

$$Y = \left(\frac{1-\theta}{\theta} \cdot \frac{p_w}{w} \right)^{1-\theta} K \quad (1)'$$

from (1) and (2).

GDP is distributed to both the real economic sector and the monetary sector. Accordingly we obtain

$$Y = \pi K + wL + \gamma Y, \quad (3)$$

where π denotes the profit rate of capital stock and γ denotes the distribution rate of GDP to the monetary sector.

We represent the consumptions of Players R and M by

$$C_R = (1 - \alpha)\pi K, \quad (4)$$

$$C_M = (1 - \beta)\gamma Y \quad (5)$$

respectively, where α and β denote the consumption properties of Player R and Player M. Moreover, α and β are the strategies of the players. Strategies α and β have the constraints; $0 \leq \alpha \leq \bar{\alpha}$, $0 \leq \beta \leq \bar{\beta}$ where $\bar{\alpha} (< 1)$ and $\bar{\beta} (< 1)$ are constants. In contrast, $\alpha\pi K$ and $\beta\gamma Y$ represent the investments of Player R and Player M in production, respectively. Therefore, we obtain

$$\dot{K} = \alpha\pi K + \beta\gamma Y - \delta K \quad (6)$$

where a dot (\cdot) denotes the derivative with respect to time and the constant δ denotes the depreciation rate of capital stock.

Next, we assume that the distribution rate of GDP to Player M, γ , is determined by

$$\gamma = \frac{\dot{Y}}{Y} + \mu. \quad (-1 \leq \mu \leq 1)^3 \quad (7)$$

where μ is a constant. That is, the distribution rate for Player M is determined by the sum of the growth rate and μ which denotes how much Player M can receive from GDP over the growth rate. As discussed below, we focus our attention on the range of μ that enables the game to have a solution

Now, we define the problems of the player as follows;

$$\max_{\alpha} \int_0^{\infty} e^{-\rho} C_R dt, \quad \text{s.t. (6), } 0 \leq \alpha \leq \bar{\alpha}, \quad (8)$$

$$\max_{\beta} \int_0^{\infty} e^{-\rho} C_M dt, \quad \text{s.t. (6), } 0 \leq \beta \leq \bar{\beta}. \quad (9)$$

where ρ is the discount rate of player times.

We complete the construction of the model by (1),(2),(3),(6),(7),(8) and(9). The seven unknowns are $Y, K, L, \pi, \alpha, \beta, \gamma$.

3. Analysis

3.1 Rearrangement of Model

We rearrange the model constructed in Section 2. Rearranging (3) and (10) using (1)', we obtain

$$\pi = \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} p_k - \left(\frac{\dot{K}}{K} + \mu \right) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}. \quad (10)$$

Since $\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K}$ by (1)', Equation (6) becomes

$$\dot{K} = MK$$

(11)

where

³ To construct the game, we assume $-1 \leq \mu \leq 1$. However, as is shown below, the same conditions for μ are required for the game to have its solution.

$$M = M(\alpha, \beta) = \frac{\{(1-\mu)\alpha + \beta\} \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \alpha \frac{1-\theta}{\theta} p_k - \delta}{1 + (\alpha - \beta) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}} . \quad (12)$$

$M = M(\alpha, \beta)$ implies that M is the function of α and β . In (12), the condition that the denominator cannot be zero is required. That is, the combination of strategies (α, β) should satisfy

$$- \alpha \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} + \beta \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \neq 1 . \quad (G0)$$

Next, let us rearrange the problems of the players. First, we define the Hamiltonian of Player R, H_R , as follows;

$$H_R = (1-\alpha) \left\{ \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} p_k - (M + \mu) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right\} K + \lambda_R M , \quad (13)$$

where λ_R is the auxiliary valuable of Player R and obeys the following equation;

$$\dot{\lambda}_R = \rho \lambda_R - \frac{\partial H_R}{\partial K} \quad (14)$$

Similarly, we define the Hamiltonian of Player M, H_M , as follows;

$$H_M = (1-\beta) (M + \mu) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} + \lambda_M M , \quad (15)$$

where λ_M is the auxiliary valuable of Player M and obeys the following equation;

$$\dot{\lambda}_M = \rho \lambda_M - \frac{\partial H_M}{\partial K} \quad (16)$$

The problem of Player R is

$$\max_{\alpha} H_R \quad \text{s.t.} \quad (6), (14), \quad 0 \leq \alpha \leq \bar{\alpha} . \quad (17)$$

Gathering the terms which include α , we get

$$\alpha \left[\left\{ \lambda_R - \left(1 - \beta \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right) \right\} \left\{ (1-\mu) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} p_k \right\} - (\delta - \mu\beta) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right] .$$

Hamiltonian H_R is a linear function of α . Therefore, the strategy of Player R is expressed

as follows;

$$\begin{cases} \lambda_R > V_R \rightarrow \alpha = \bar{\alpha}, \\ \lambda_R = V_R \rightarrow ???, \\ \lambda_R < V_R \rightarrow \alpha = 0. \end{cases} \quad (18)$$

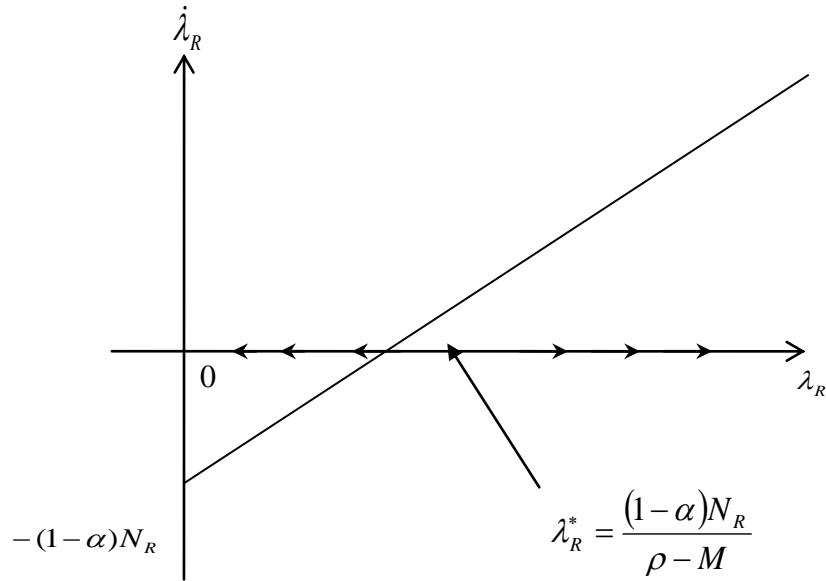
where

$$V_R = V_R(\beta) = \frac{(\delta - \mu\beta) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}}{(1-\mu) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} p_k} + \left\{ 1 - \beta \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right\}. \quad (19)$$

The point whose value is equal to V_R is called **the strategy exchange point of Player R**.

Figure 1 Path of the auxiliary valuable λ_R

(a) the case of $M < \rho$



$$N_R = \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} p_k - (M + \mu) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}$$

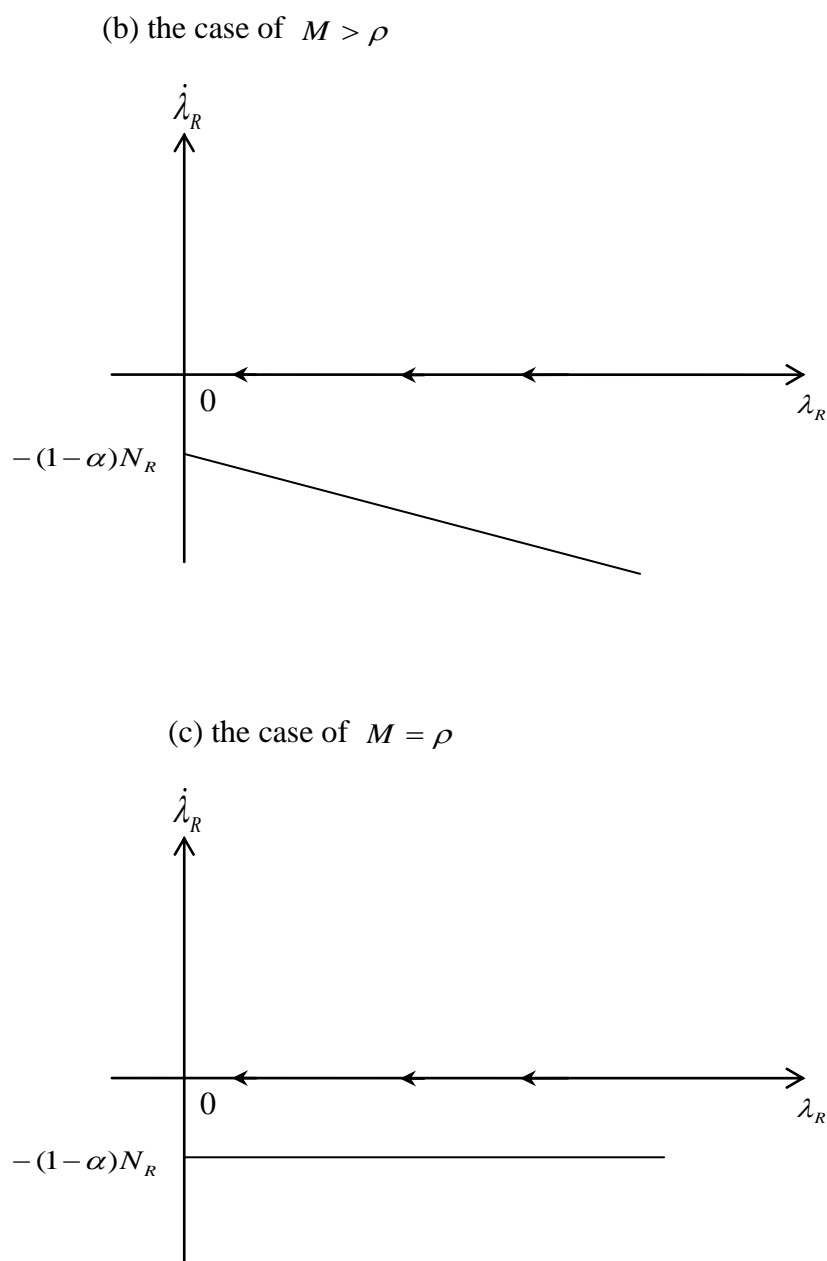


Figure-1 illustrates the paths of the auxiliary valuable λ_R which obeys Equation.(14).

From Figure-1, $M(\bar{\alpha}, \bar{\beta})$ should satisfy the condition;

$$M(\bar{\alpha}, \bar{\beta}) \geq \rho, \quad (\text{M1})$$

because $M(\bar{\alpha}, \bar{\beta}) < \rho$ implies that the economy has no power for decreasing the value of the auxiliary valuable λ_R in terms of capital accumulation. Therefore, we assume that (M1) is satisfied in our game.

In contrast, the problem of Player M is

$$\max_{\beta} H_M \quad \text{s.t.} \quad (6), (16), \quad 0 \leq \beta \leq \bar{\beta}. \quad (20)$$

Gathering the terms which include β , we get

$$\beta \left[\lambda \mu - \alpha \left\{ \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} p_k \right\} - (\mu - \delta) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right]$$

Hamiltonian H_M is a linear function of β . Therefore, the strategy of Player M is expressed as follows;

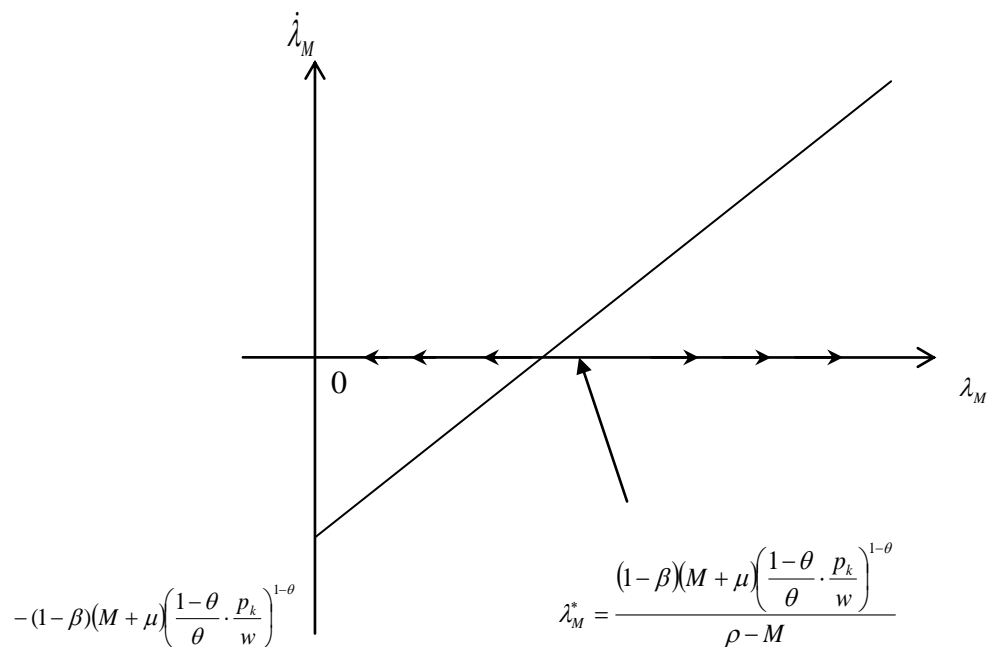
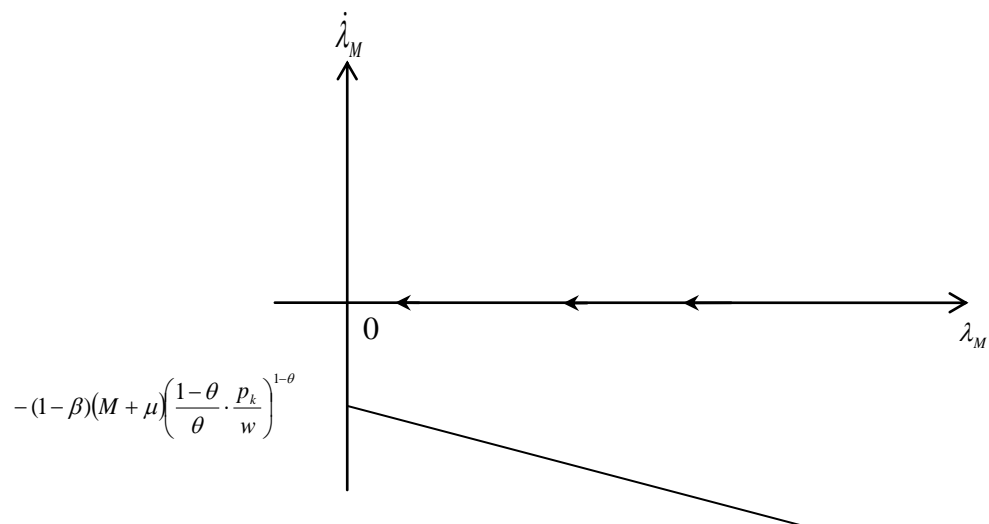
$$\begin{cases} \lambda_M > V_M \rightarrow \beta = \bar{\beta}, \\ \lambda_M = V_M \rightarrow ??, \\ \lambda_M < V_M \rightarrow \beta = 0. \end{cases} \quad (21)$$

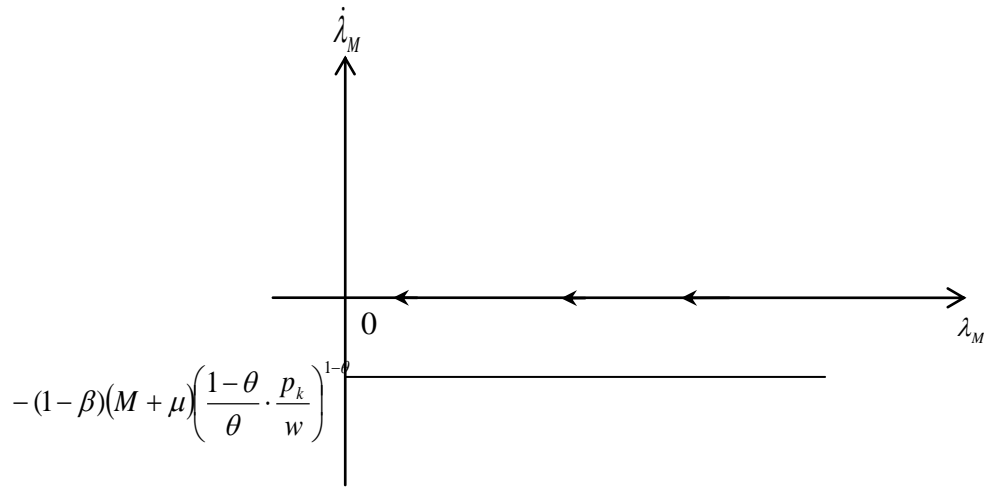
where

$$V_M = V_M(\alpha) = \frac{\alpha \left\{ \left(\frac{1-\theta}{\theta} \right)^{1-\theta} - \frac{1-\theta}{\theta} p_k \right\} - \delta}{\mu} + 1 \quad (22)$$

The strategy exchange point of Player M is hereby defined as the point whose value is equal to V_M .

Figure-2 illustrates the path of the auxiliary valuable λ_M which obeys Equation.(16).

Figure 2 Path of the auxiliary valuable λ_M (a) the case of $M < \rho$ (b) the case of $M > \rho$ (c) the case of $M = \rho$



3.2 Analysis of the Game

At this stage, we propose the condition for the game to have a solution. First, we obtain the following proposition regarding $M = M(\alpha, \beta)$.

Proposition1: For the game to have a solution, the following conditions for μ are necessary. That is,

$$\mu \geq \delta, \quad (G1)$$

$$\frac{\bar{\alpha} \left\{ (1-\rho) \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} \cdot p_k \right\} - (\rho + \delta)}{\bar{\alpha} \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}} < \mu < \frac{\rho \left\{ 1 - \bar{\beta} \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} \right\} + \delta}{\bar{\beta} \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}} \quad (G2).$$

(pf.) (G1); If $\alpha = 0$ in (22), then $V_M(0) = \frac{\mu - \delta}{\mu}$. The condition $V_M(0) \geq 0$ is required.

(G2) ; (G2) is obtained by the following two conditions;

$$M(\bar{\alpha}, 0) < \rho, \quad (M2)$$

$$M(0, \bar{\beta}) < \rho. \quad (M3)$$

These are also the conditions needed for the game to have a solution. Let us consider (M3). If $M(0, \bar{\beta}) \geq \rho$, then Player R cannot change the situation of $M(\alpha, \bar{\beta}) \geq \rho$ for all α .

Therefore, Figures-1(b) or 1(c) occurs. If the value of λ_R sufficiently decreases and

becomes smaller than the value of $V_R(\bar{\beta})$ or $\lambda_R^*(\bar{\alpha}, 0)$ which is the value of the steady

state of Eq.(14), then the value of λ_R will become negative in the future and the game will become meaningless. That is, $M(0, \bar{\beta}) \geq \rho$ implies that Player M has a strong strategy which renders the strategy of Player R meaningless.

Similarly, we can judge $M(\bar{\alpha}, 0) < \rho$. (Q.E.D)

With respect to $M(\bar{\alpha}, 0)$ and $M(0, \bar{\beta})$, we should consider whether their values are positive. For instance, $M(\bar{\alpha}, 0) \leq 0$ implies that Player R cannot make capital accumulation become positive by his (or her) own efforts. Indeed, even under such a condition, the game may have a solution, which is constructed below for some restricted conditions. However, the condition where players cannot accumulate capital by his (or her) own efforts is meaningless from a game-theoretical viewpoint. Therefore, we state the following assumptions.

Assumption

$$M(\bar{\alpha}, 0) > 0, \quad (\text{M4})$$

$$M(0, \bar{\beta}) > 0. \quad (\text{M5})$$

From (M4) and (M5), we obtain

$$\frac{\delta}{\bar{\beta} \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}} < \mu < \frac{\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} \cdot p_k - \frac{\delta}{\bar{\alpha}}}{\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta}}. \quad (\text{G3})$$

In keeping with the above preparations, let us examine the game's solution. As shown in Figure-1 and Figure-2, the auxiliary equations (14) and (16) may have steady states in some cases. However, they are unstable. Therefore, we cannot adopt the ordinary method for stability analysis. To construct the game's solution, we should therefore solve the following two problems.

- (I) The first problem is to search for the strategy combination (α, β) that satisfies Equations (17) and (20).

(II) The second is to investigate the condition for strategy combination (α, β) to satisfy its constraints.

If they are typical optimal control problems, these problems must be solved together. However, the steady state in our game is unstable. Therefore, we analyse them individually.

3.3 Instability of the Capitalist Economy

Let us analyse problem (I). As mentioned above, the game does not any stable steady points. Therefore, we can guess that if the solution to the game exists, players must obey the following strategy rule.

⟨Strategy Rule⟩

The player can use an arbitrary strategy if the value of the auxiliary variable is in the strategy exchange point [see (18) and (21)]. Let us consider the time when the auxiliary variable reaches the strategy exchange point. If it occurs at Player R's point, Player R takes strategy $\hat{\alpha}$ such that $V_R(\beta) = \lambda_R^*(\hat{\alpha}, \beta)$ for a given β , and if it occurs at Player M's point, then Player M takes the strategy $\hat{\beta}$ such that $V_M(\alpha) = \lambda_M^*(\alpha, \hat{\beta})$ for a given α .

Note that the *strategy rule* has meaning, only when $M(\hat{\alpha}, \beta) < \rho$ for a given β and $M(\alpha, \hat{\beta}) < \rho$ for a given α . If these conditions are not satisfied, the auxiliary equations do not have any steady state points [see Figure-1(a) and Figure-2(a)]. Therefore, we assume

$$M(\hat{\alpha}, \bar{\beta}) < \rho, \quad (M6)$$

$$M(\bar{\alpha}, \hat{\beta}) < \rho. \quad (M7)$$

The title of this section is 'Instability of the Capitalist Economy'. Accordingly, let us examine what happens in the economy if players obey the *strategy rule*. We conduct step-by-step analysis.

Step1. Players implement the strategies corresponding to (18) and (21) for the initial values of the auxiliary variables.

Step 2. The first player whose auxiliary variable reaches the strategy exchange point obeys *the strategy rule*. Let us assume Player M reaches his (or her) strategy exchange point at first. At this moment, Player M takes the strategy that makes the strategy exchange point coincide with the steady state of his (or her) auxiliary equation.

Step 3. Through the exchange of Player M's strategy in Step 2, the positions of the strategy exchange point and the steady state point of Player R shift. However, when the value of the auxiliary variables of Player R reaches his(or her) strategy exchange point, Player R follows the strategy given by the *strategy rule*. That is, the strategy exchange point coincides with the steady state.

Step 4 Through the exchange of the Player R's strategy in Step 3, the positions of the strategy exchange point and the steady state point of Player M which were set so as to coincide in Step 2 shift. However, when the value of the auxiliary variables of Player M reaches his(or her) strategy exchange point, Player M decides upon a strategy as determined *by the strategy rule*.

Step 5 Each player continues to follow the strategy shown in the above steps in perpetuity.

There is an important and serious problem in Steps 1-5. That is, when one player exchanges his(or her) strategy by the *strategy rule*, the other player's strategy exchange point and steady state point shift. However, there is no guarantee that the value of auxiliary variable of the other player necessarily reaches his(or her) strategy exchange point subsequently.

Let us analyse this point. For instance, the strategy exchange point of Player R coincides with his(or her) steady state in Step 3. When Player M exchanges his(or her) strategy in Step 4, the strategy exchange point and steady state point of Player R shift. In this case, the value of the auxiliary variable reaches his (or her) strategy exchange point subsequently. This is because the strategy exchange point and the steady state point of Player R move in reverse directions[See (19) and λ_R^* in Figure-1(a)]. This situation is depicted in Figure-3. Therefore, the old strategy exchange point converges to the new

one.

Figure 3 Position change of Player R when β increases

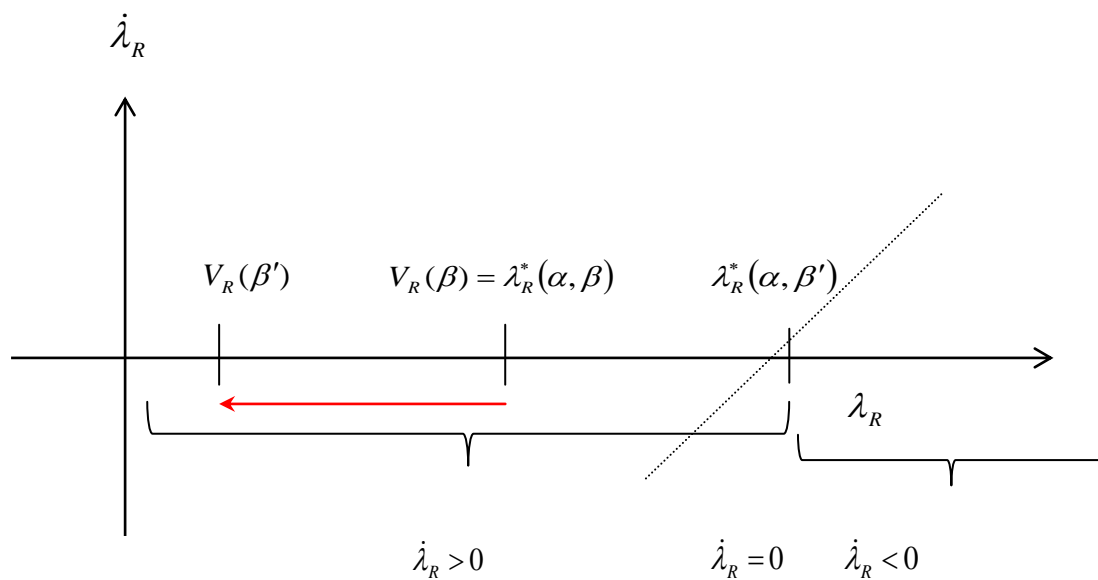
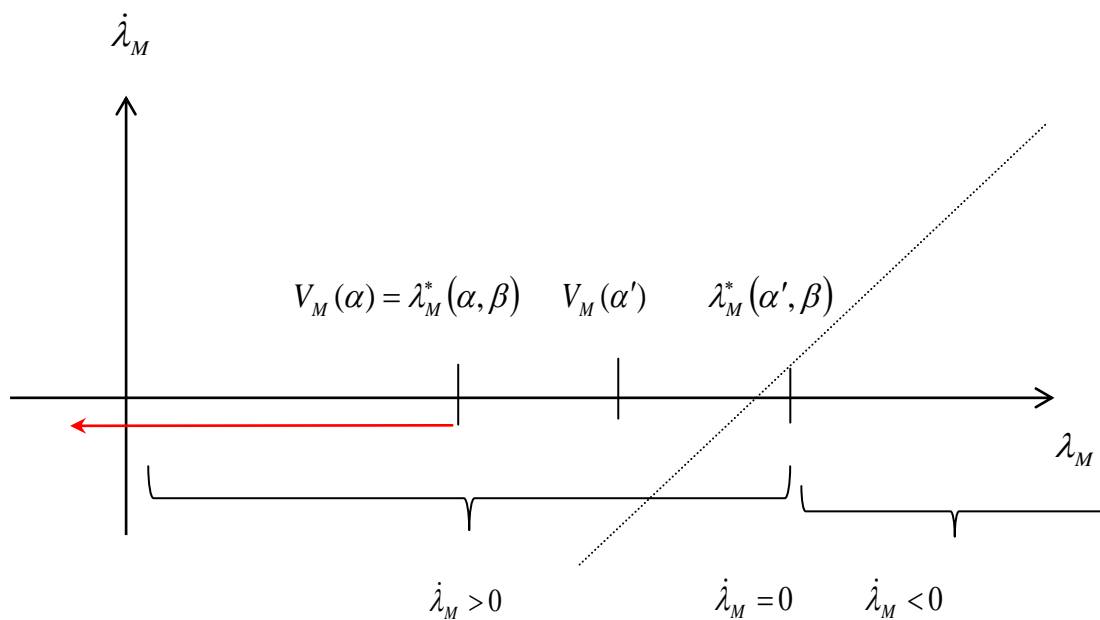


Figure 4 Position change of Player M when α increases



Next, let us analyse Step3 and 4. In Step 3, Player R exchanges his(or her)strategy and follows a strategy as determined by the *strategy rule*. Through this exchange of strategy, the strategy exchange point and the steady state point which were in the same position shift. However, they move in same direction. See (22) and λ_M^* in Figure-2(a). This situation, depicted in Figure-4, indicates that the old strategy exchange point does not converge to the new one and the value of the auxiliary variable becomes negative, either sooner or later. Therefore, players cannot continue playing the game and Step 5. We should indicate that this phenomenon implies the instability of the capitalist economy.

3.4 Political Aid for the game to have its solution

Therefore, solving the problem requires political aid to conquer this instability. This section proposes the political method that conquers our problem.

⟨Policy Maker and Policy Rule⟩

We assume that a policy maker exists and can control μ which represents how much rate Player M can receive over the growth rate [see (7)]⁴.

Assume that Player M's strategy exchange point and steady state point are in the same position at a certain time. Next, assume Player R exchanges his(or her) strategy. Thereafter, Player M's strategy exchange point and steady state point shift. As mentioned above, the two points move in the same direction. This fact causes the economy's instability. Therefore, the policy maker controls μ in order for Player M's strategy exchange point to shift in a different direction than that of the steady state point. That is, if Player R increases the value of his (or her) strategy α , the policy maker should decrease the value of μ by a sufficient amount and vice versa[see (22)].

Given the existence of policy maker and his(or her) policy rule, we propose the following.

⁴ As indicated in Section 4, the policy maker is considered to be a central bank. Therefore, its method is financial policy, especially open market operations, that controls the quantity of money and the interest rate. See (28) in Section 4.

Proposition 2: The solution of optimal control

For the strategy combination (α, β) and parameter μ , we assume that (G1),(G2) and (G3) are satisfied. We also assume the existence of policy maker. He (or she) can control μ by adopting the *policy rule* under (G1),(G2),and (G3).

For initial conditions of the auxiliary variables λ_R^0 and λ_M^0 , we assume the following conditions; (C1) and (C2).

$$\lambda_R^*(0, \bar{\beta}) \leq \lambda_R^0 \leq \lambda_R^*(\bar{\alpha}, 0), \quad (C1)$$

$$\lambda_M^*(\bar{\alpha}, 0) \leq \lambda_M^0 \leq \lambda_M^*(0, \bar{\beta}). \quad (C2)$$

Thereafter, there exists a strategy pair $(\alpha(t), \beta(t))$ that satisfies Equations (17) and (19) with the aid of the policy rule.

(pf.) Let us consider the condition of the left side of (C1), using Figure-5. Note that $\lambda_R^*(0, \bar{\beta}) \leq V_R(\bar{\beta})$, because $\alpha = 0$. If the left side of (C1) is not satisfied and Player M takes the strategy $\beta = \bar{\beta}$, then Player R cannot exchange strategies and the value of λ_R becomes negative. In this case, the game does not have a solution. Therefore, the left side of (C1) should be necessary. We can analyse the left side of (C2) similarly.

Next, we consider the right side of (C1). $V_R(0) \leq \lambda_R^*(\bar{\alpha}, 0)$, because $\alpha = \bar{\alpha}$. If the right inequality or equality of (C1) is not held and Player M takes the strategy $\beta = 0$, then Player R cannot swap his (or her) strategy and the value of λ_R becomes $+\infty$. In this case, the game does not have a solution. Therefore, the right side of (C1) is required. We assume the right of (C2) is needed according to the same logic.

Figure-5 Exchange points of the strategy of Player R and his(her) strategy

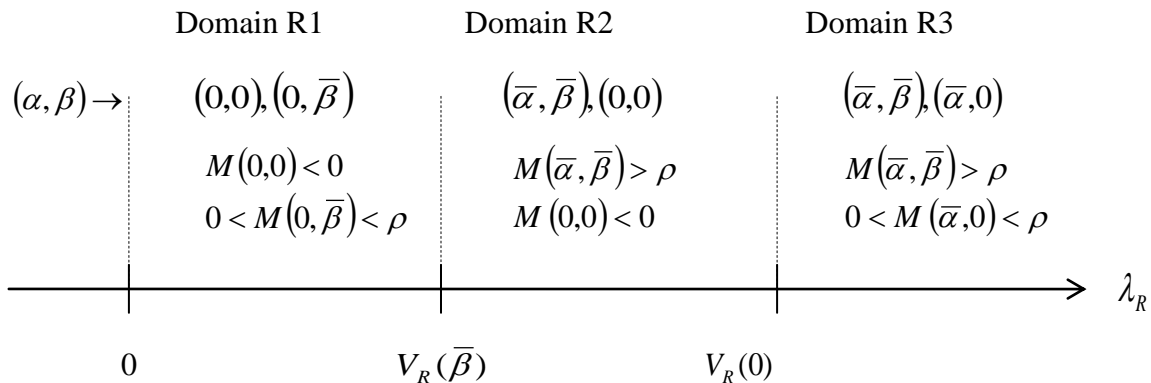
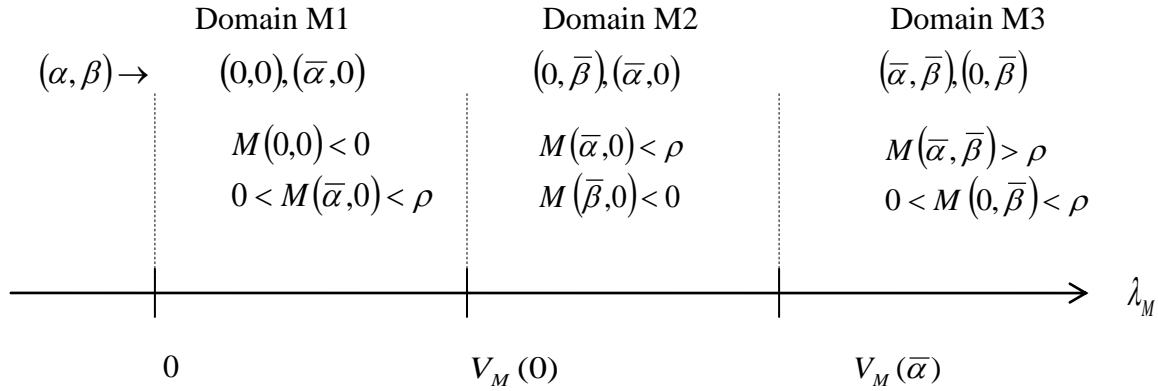


Figure-6 Exchange points of the strategy of Player M and his(her) strategy

Note that (C1) defines the upper limit of $\bar{\beta}$ and the lower limit of $\bar{\alpha}$ and (C2) defines the upper limit of $\bar{\alpha}$ and the lower limit of $\bar{\beta}$.

Now let us construct the solution to the game by using some examples. This construction seems to be arbitrary. However, under complete information, players should search for the strategy combination (α, β) that satisfies Equations (17) and (19). Under complete information, players recognize that they should obey the *strategy rule*, if not, they cannot construct optimal strategies.

First, we consider the case where the auxiliary variables' initial values are in the domain of R3 and M3. Since Player R and Player M choose $\alpha = \bar{\alpha}$ and $\beta = \bar{\beta}$, respectively, then $M(\bar{\alpha}, \bar{\beta}) > \rho$ and Figure-1 (b) and Figure-2 (b) are chosen. Therefore, the values of λ_R and λ_M decrease. In this case, the value of either λ_R or λ_M reaches the strategy exchange point, either first or simultaneously. In the former case, if it is the point of Player R, he (or she) takes the strategy $\hat{\alpha}$, which satisfies $V_R(\bar{\beta}) = \lambda_R^*(\hat{\alpha}, \bar{\beta})$, or if it is the point of Player M, he (or she) takes the strategy $\hat{\beta}$, which satisfies $V_M(\bar{\alpha}) = \lambda_M^*(\bar{\alpha}, \hat{\beta})$. Note that the auxiliary equations have steady state points which satisfy (M6) and (M7).

Let us analyse the case where Player M reaches his (or her) strategy exchange point first. We denote the time when Player M reaches his (or her) strategy exchange point by \hat{t} . At time \hat{t} , Player M exchanges his (or her) strategy from $\beta = \bar{\beta}$ to $\beta = \hat{\beta} (\leq \bar{\beta})$. Therefore, the strategy exchange point of Player R shifts from $V_R(\bar{\beta})$ to $V_R(\hat{\beta}) (\geq V_R(\bar{\beta}))$ [see (19)].

Thereafter, for the relative positional relation between $V_R(\hat{\beta})$ and $\lambda_R(\hat{t})$, the following three cases could happen. That is, (i) $V_R(\hat{\beta}) < \lambda_R(\hat{t})$, (ii) $V_R(\hat{\beta}) > \lambda_R(\hat{t})$ and (iii) $V_R(\hat{\beta}) = \lambda_R(\hat{t})$.

In the case of (i), Player R continues to take the strategy $\alpha = \bar{\alpha}$. By (M6), it happens such that $M(\bar{\alpha}, \hat{\beta}) < \rho$. In addition, by (C3), it happens such that $V_R(\hat{\beta}) < \lambda_R(\hat{t}) < \lambda_R^*(\bar{\alpha}, \hat{\beta})$.

Therefore, the value of the auxiliary variable λ_R decreases and will reach $V_R(\hat{\beta})$. At the moment when the value of λ_R reaches $V_R(\hat{\beta})$, Player R takes the strategy α' which satisfies $V_R(\hat{\beta}) = \lambda_R^*(\alpha', \hat{\beta})$ according to *the strategy rule*.

At this moment, Player R changes his(or her) strategy from $\bar{\alpha}$ to $\alpha' (< \bar{\alpha})$. Therefore, the values of $V_M(\bar{\alpha})$ and $\lambda_M^*(\bar{\alpha}, \hat{\beta})$ decrease to $V_M(\alpha')$ and $\lambda_M^*(\alpha', \hat{\beta})$ respectively. We cannot judge which value is greater, $V_M(\alpha')$ or $\lambda_M^*(\alpha', \hat{\beta})$. However, since $V_M(\alpha')$ and $\lambda_M^*(\alpha', \hat{\beta})$ are smaller than $V_M(\bar{\alpha}) = (\lambda_M(\hat{t}))$, then $\lambda_M(t) \rightarrow \infty (t \rightarrow \infty)$ and the game is destroyed. Therefore, the policy maker should control μ by *the policy rule*. That is, he (or she) controls μ so as for $V_M(\alpha')$ to increase. Thereafter, the value of $\lambda_M(t)$ moves to $V_M(\alpha')$. At the moment when the values of $\lambda_M(t)$ and $V_M(\alpha')$ coincide, Player M obeys the *strategy rule*.

Next, let us analyse case (ii), where Player R swaps his (or her) strategy α from $\alpha = \bar{\alpha}$ to $\alpha = 0$ at time \hat{t} . At this moment, it happens $M(0, \hat{\beta}) < \rho$ and $\lambda_R^*(0, \hat{\beta}) \leq \lambda_R^*(0, \bar{\beta}) < V_R(\bar{\beta}) < \lambda_R(\hat{t})$. Therefore, the value of the auxiliary variable λ_R increases and will reach $V_R(\hat{\beta})$. At the moment when the value of λ_R reaches $V_R(\hat{\beta})$, Player R adopts the strategy α'' that satisfies $V_R(\hat{\beta}) = \lambda_R^*(\alpha'', \hat{\beta})$ according to *the strategy rule*.

Player M's reaction to Player R's exchange of strategy and the support of the policy maker are the same as those in case (i).

Finally, let us consider case (iii). In this case, Player R takes the strategy α''' which satisfies $V_R(\hat{\beta}) = \lambda_R^*(\alpha''', \hat{\beta})$ according to *the strategy rule*. Player M's reaction to Player R's exchange of strategy and the support of the policy maker are the same as those in case (i).

The players only have to repeat the above process. Note that the values of the auxiliary variables are confined to the finite domains, $(\lambda_R^*(0, \bar{\beta}), \lambda_R^*(\bar{\alpha}, 0))$, $(\lambda_M^*(\bar{\alpha}, 0), \lambda_M^*(0, \bar{\beta}))$. Indeed, the case may occur where players reach the strategy (α^*, β^*) that satisfies

$$\begin{cases} V_R(\beta^*) = \lambda_R^*(\alpha^*, \beta^*), \\ V_M(\alpha^*) = \lambda_M^*(\alpha^*, \beta^*) \end{cases} \quad (23)$$

In either case, the transversal conditions are satisfied.

We have analysed the case where the initial values of the auxiliary variables are in the domain R3 and M3. However, even if these initial values are in other domains, we can construct the strategy pair that satisfies the optimal problems.

(QED)

Next, let us analyse the problem (II). The strategy constructed in proposition 2 should satisfy the following conditions;

$$\begin{cases} 0 < \alpha^* < 1, \\ 0 < \beta^* < 1, \\ 0 < M(\alpha^*, \beta^*) < \rho. \end{cases} \quad (24)$$

With respect to (24), we can propose the following proposition.

Proposition 3 If μ satisfies

$$\mu < \frac{\rho \left[\alpha \left\{ \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \frac{1-\theta}{\theta} \cdot p_k \right\} - \delta \right]}{(1-\beta) \left\{ M(\alpha^*, \beta^*) + \mu \left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w} \right)^{1-\theta} - \rho \right\}}, \quad (G4)$$

and the solutions of (23), α^* and β^* , take positive values, then the strategy combination

(α, β) as constructed in proposition 2 satisfies Equation (24).

(pf.) First, we write out the terms in (23) concretely.

$$\begin{aligned}
V_R(\beta) &= \frac{\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} (\delta - \beta\mu)}{(1-\mu)\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} - \frac{1-\theta}{\theta} \cdot p_k} + \left\{1 - \beta\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta}\right\} \\
\lambda_R^*(\alpha, \beta) &= \frac{(1-\alpha)\left[\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} - \frac{1-\theta}{\theta} \cdot p_k - \{M(\alpha, \beta) + \mu\}\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta}\right]}{\rho - M(\alpha, \beta)} \\
V_M(\alpha) &= \frac{\alpha\left\{\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} - \frac{1-\theta}{\theta} \cdot p_k\right\} + (\mu - \delta)}{\mu} \\
\lambda_M^*(\alpha, \beta) &= \frac{(1-\beta)\{M(\alpha, \beta) + \mu\}\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta}}{\rho - M(\alpha, \beta)}
\end{aligned}$$

Except $1 + (\alpha - \beta)\left(\frac{1-\theta}{\theta} \cdot \frac{p_k}{w}\right)^{1-\theta} = 0$ [see (G0)], the function $M(\alpha, \beta)$ is continuous. In addition, we assume (M6) and (M7). Therefore, the above four functions are continuous in the domain of the definition we consider. Thus, it is apparent that the equation system (23) has a solution.

Let us now analyse the condition (23). Since $V_M(\alpha) > 0$, we get $M(\alpha^*, \beta^*) < \rho$ by the second equation of (23). In addition, we obtain $M(\alpha^*, \beta^*) > 0$ by (G4).

On the other hand, we get $\alpha^* < 1$ and $\beta^* < 1$ by $V_R(\beta^*) = \lambda_R^*(\alpha^*, \beta^*) > 0$ and $V_M(\alpha^*) = \lambda_M^*(\alpha^*, \beta^*) > 0$.

The remaining problem is the positivity of α^* and β^* . It depends on the value of the parameter μ . We can insist that if we place an additional condition, (G4), we can guarantee the positivity of α^* and β^* . (Q.E.D.)

We can guarantee the existence of a solution to the game by proposition 2 and proposition 3.

Finally, we should insist on two points. First, since the auxiliary equation of our game has no stable steady point, we cannot adopt the usual method for analysing the optimal problem. Under complete information, players know these conditions and construct their strategies to enable the optimal conditions to be satisfied.

Second, our game needs the aid of financial policy as indicated in the *policy rule*. This implies that the capitalist economy is unstable if left to itself and needs the proper control of μ .

3.5 Definition of the Natural Interest Rate

Let us consider Equation (23). It indicates that the strategy exchange point of each player coincides with the steady state point. As indicated in Equations (18) and (21), players invest all of their savings (that is, remaining part of consumption) or nothing except the strategy exchange points. They hope to improve the condition of the economy. However, players think that the strategy exchange points are neutral, hence they feel no need to improve their condition. The word ‘neutral’ means that consumption and investment are *indifferent* for players. In addition, players’ savings are equal to the investment that contributes to capital accumulation [see (6)]. These conditions construct the character of the *natural economy*. Therefore, the strategy combination (α^*, β^*) is called the *natural strategy* and the growth rate of economy that is accomplished by the natural strategy is called the *natural growth rate*. For the *natural strategy*, the economy is considered as being in the stable steady point.

On the other hand, we define the *natural distribution rate* of Player M, γ^* , by

$$\gamma^* = M(\alpha^*, \beta^*) + \mu^*, \quad (25)$$

where μ^* satisfies (G0),(G1)~(G4).

Next, let us state

$$rM = \gamma Y, \quad (26)$$

where M and r denote the amount of monetary assets and its interest rate respectively.

We define the *natural nominal interest rate* r^* by

$$r^* M = \gamma^* Y^* \quad (27)$$

where Y^* denotes the GDP produced by the *natural strategy*. Since we define the *natural nominal interest rate* as that which depends on the amount of monetary assets, we also call the *natural distribution rate*, γ^* , the *natural real interest rate*. By definition, if $M = Y$, then the *natural nominal interest rate* is equal to the *natural distribution rate* (the *natural real interest rate*).

As mentioned in the introduction, a range of definitions have been offered for the term *natural economy*. However, it is important to recognize that the *natural economy* is an absolute criterion from which the economy cannot deviate over a certain range. We should recognize that the large deviation from the *natural rate* can serve as an important alarm.

4. Fundamental Cause of Financial Crisis

In Section 3, we analysed the condition for the game to have a solution. By this analysis, we obtain the result that rigorous conditions exist that μ should satisfy for the capitalist economy to be stable.

In this section, we investigate the actual conditions with respect to μ . We select three areas to serve as case studies, namely, USA, Euro Area and Japan. Our purpose is to indicate what happens regarding μ and to prove that imbalance between the real economic sector and monetary sector caused the economic crisis.

4.1 Preparation for the Simulation

To introduce the real economy to our game, let us state

$$M = aY, \quad (28)$$

where M denotes the amount of the financial assets held by financial corporations. The parameter a is the ratio of M to GDP.

From Equations (26), (28) and (7), we obtain

$$\mu = ar - \frac{\dot{Y}}{Y}. \quad (29)$$

Now, we try a simulation of Equation (29).

4.2 Current Trends and our Model

First, we should confirm current trends of the actual economy. The remarkable trend in the current economy is the rapid increase in its monetary base. This trend is common in USA, Euro Area and Japan. As is well known, this is the result of Keynesian fiscal and financial policies. Consequently, the financial assets rapidly expanded and the scale of the world's financial assets became 3.5 times as large as that of the world's GDP⁵. We should indicate that this fact implies a large deviation from the *natural economy*. Therefore we should investigate what happened regarding μ behind the financial crisis such as the dot-com bubble, the Lehman crash or PIIGS.

Now we demonstrate the transition of μ and the underlying data used in calculating μ for USA, Euro Area and Japan.

In the Euro area, the ratio of financial assets to GDP is increasing and reached 5.9 in 2013. The ratio doubled during the past 14 years. This level seems to have reached a dangerous point. Compare it with the growth rate in the other two areas. In Figure-7(c), μ has two peaks. The increase in μ during the period of 1999-2000 corresponds to the dot-com bubble and that in 2005-2007 corresponds to the Lehman crash. These increases imply that the monetary sector absorbed the value from the real economic sector at a high rate. Consequently, μ decreased rapidly. These decreases correspond to the economic recession [see Figure-7(a)]. The further decrease in 2010 corresponds to the big recession, or PIIGS.

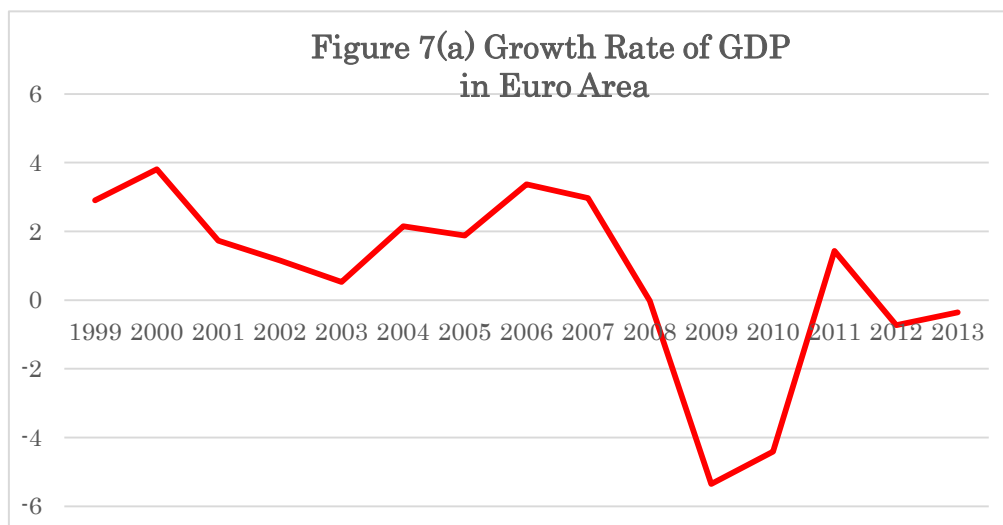
Table 1 μ in the Euro Area

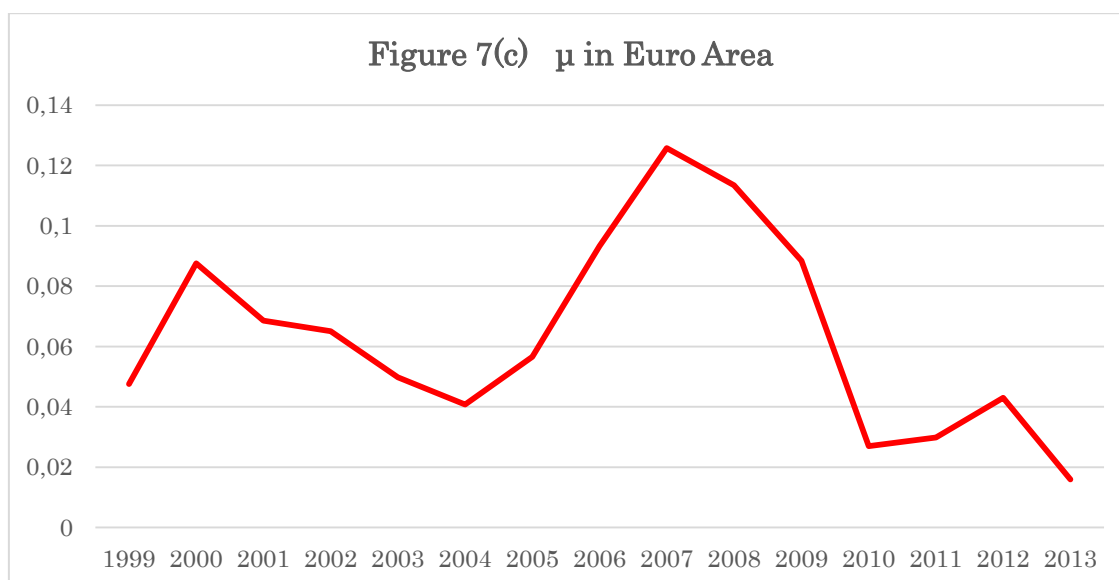
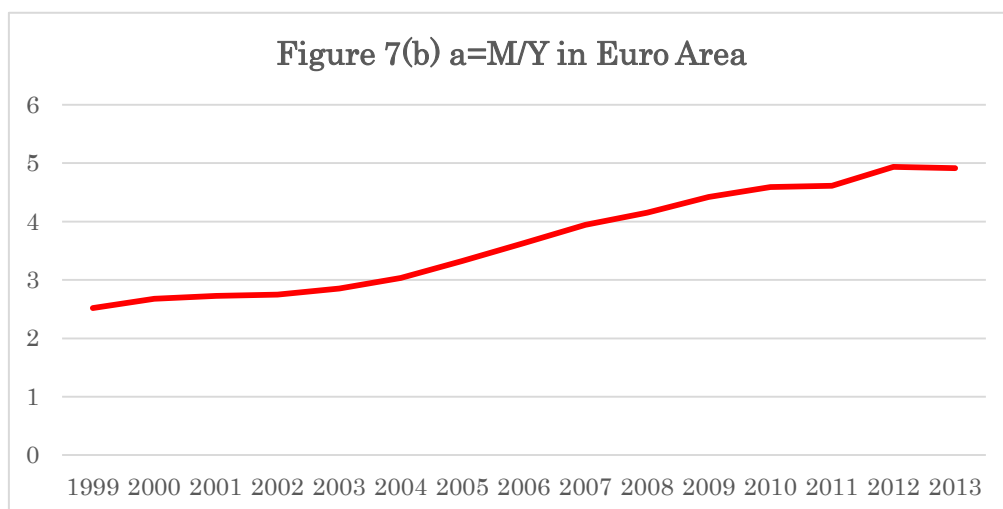
year	Growth Rate of GDP(Q3)	a=M/Y	ar	μ
1999	2.9	2.518541	0.075556	0.04754
2000	3.81	2.675116	0.127068	0.087587
2001	1.73	2.725868	0.088591	0.068591
2002	1.15	2.751174	0.074282	0.065098

⁵ <http://www.meti.go.jp/report/tsuhaku2008/2008honbun/html/i1120000.html>

2003	0.53	2.85313	0.057063	0.049767
2004	2.15	3.035233	0.060705	0.040793
2005	1.88	3.323604	0.074781	0.056607
2006	3.37	3.63114	0.12709	0.093341
2007	2.97	3.942432	0.157697	0.125757
2008	-0.02	4.149753	0.114118	0.113503
2009	-5.35	4.420957	0.04421	0.088474
2010	-4.41	4.589204	0.045892	0.02695
2011	1.43	4.611803	0.046118	0.029861
2012	-0.73	4.934687	0.03701	0.043045
2013	-0.36	4.914514	0.012286	0.016

Source; <http://www.ecb.europa.eu/home/html/index.en.html>





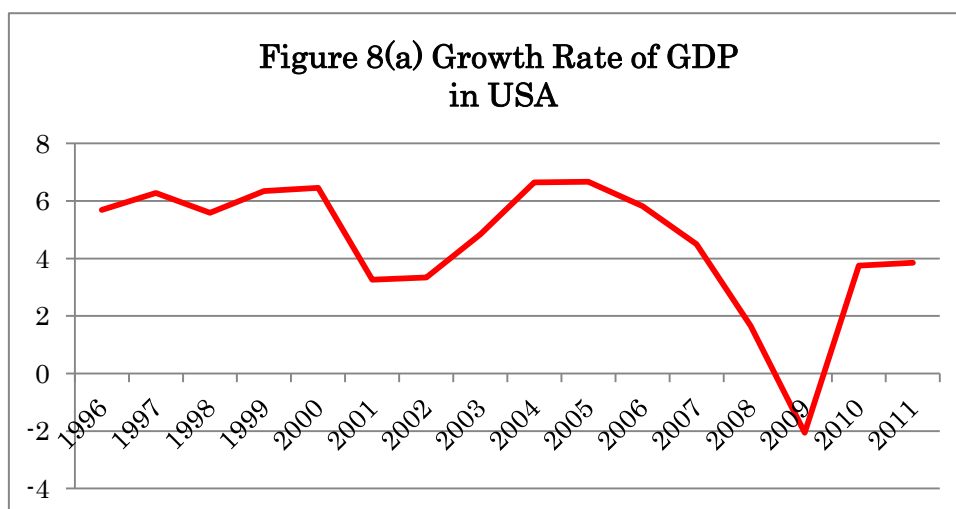
Next, let us investigate the data from USA. The graph of μ in USA also has two peaks. These peaks are sharper than those in the Euro area. However, the transitions of data in USA have the same trends as the Euro area.

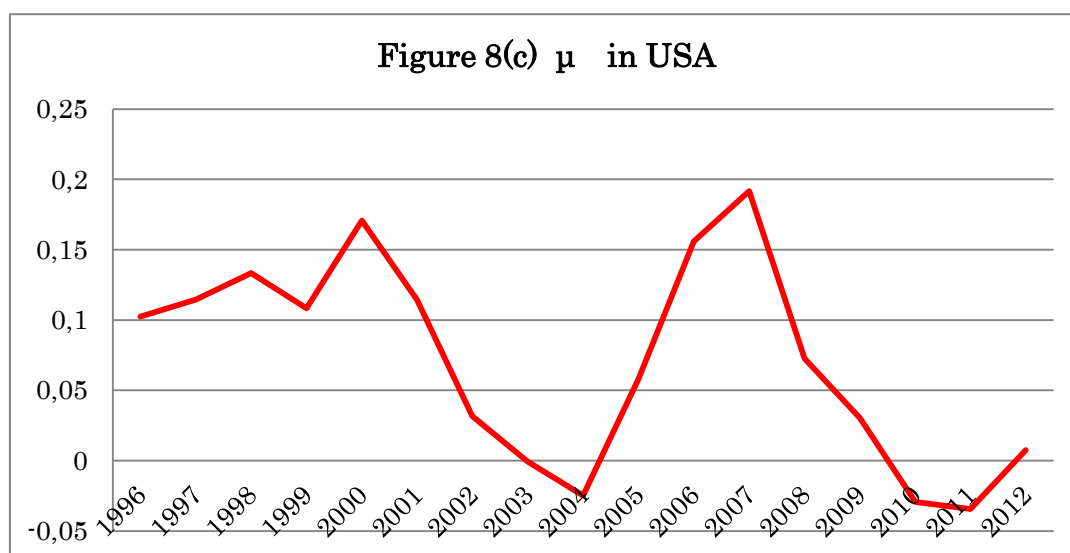
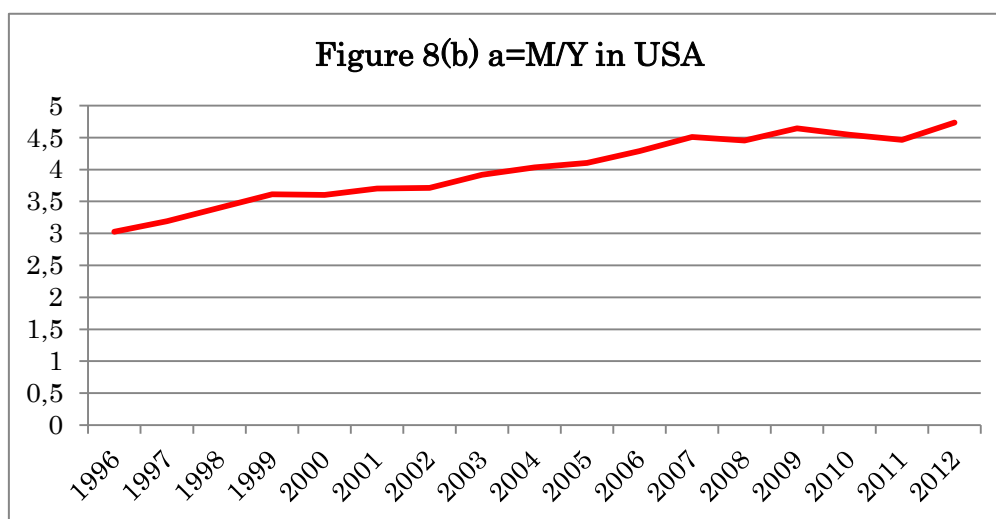
Table 2 μ in USA

year	Growth Rate of GDP(%)	$a=M/Y$	a_r	μ
1996	5.691544885	3.024098	0.15937	0.102455
1997	6.2751537	3.18922	0.177321	0.114569
1998	5.582854156	3.403109	0.189213	0.133384
1999	6.343862429	3.61053	0.171861	0.108423

2000	6.455817996	3.604031	0.235343	0.170785
2001	3.261513941	3.704366	0.147063	0.114448
2002	3.340140984	3.714313	0.065	0.031599
2003	4.845084789	3.916306	0.047779	-0.00067
2004	6.643387016	4.03531	0.041564	-0.02487
2005	6.666123646	4.104502	0.124777	0.058116
2006	5.822655284	4.29026	0.214084	0.155857
2007	4.491300991	4.508443	0.236693	0.19178
2008	1.657424225	4.45351	0.08907	0.072496
2009	-2.054305958	4.646086	0.009757	0.0303
2010	3.748118658	4.544032	0.008179	-0.0293
2011	3.847362334	4.464381	0.004018	-0.03446
2012	0	4.73501	0.007576	0.007576

Source; <http://www.federalreserve.gov/>



Table 3 μ in Japan

year	Growth Rate of GDP(%)	$a=M/Y$	ar	μ
1981	4.17684392	2.99706671	0.18731667	-0.04176844
1982	3.37660646	3.24407638	0.1784242	0.144658136
1983	3.06073847	3.55714537	0.195643	0.16503561
1984	4.4638997	3.79647788	0.18982389	0.145184897
1985	6.33335609	4.06762541	0.20338127	0.14004771
1986	2.83107728	4.67646351	0.16367622	0.13536545
1987	4.1074262	5.06372515	0.12659313	0.085518867
1988	7.14669468	5.32939771	0.13323494	0.061767996

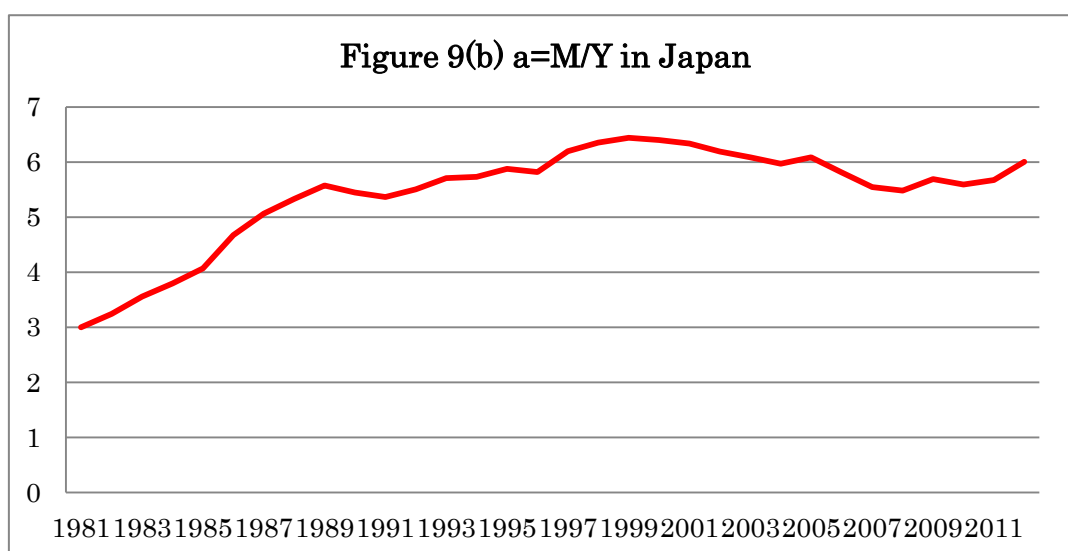
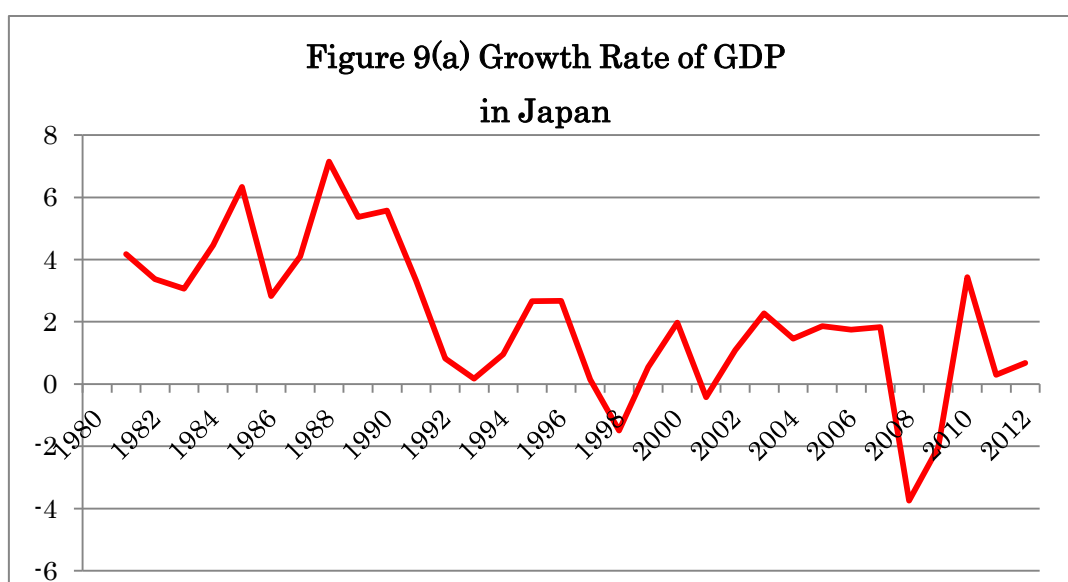
1989	5.37015049	5.57642129	0.18123369	0.127532187
1990	5.57240053	5.44944872	0.28609606	0.230372052
1991	3.32433942	5.36811761	0.32208706	0.288843662
1992	0.81902913	5.50726303	0.20652236	0.198332072
1993	0.17106356	5.70931371	0.14273284	0.141022207
1994	0.95105901	5.73140551	0.1002996	0.090789006
1995	2.65900421	5.87974167	0.05879742	0.032207375
1996	2.66933823	5.81736894	0.02908684	0.002393462
1997	0.14726994	6.19459413	0.03097297	0.029500271
1998	-1.4904699	6.3543703	0.03177185	0.046676551
1999	0.53996986	6.44011405	0.03220057	0.026800872
2000	1.9770211	6.40237555	0.03201188	0.012241667
2001	-0.4274807	6.33453631	0.01583634	0.020111148
2002	1.09238666	6.19011326	0.00619011	-0.00473375
2003	2.26833973	6.08920952	0.00608921	-0.01659419
2004	1.45830137	5.96891764	0.00596892	-0.0086141
2005	1.8568319	6.08485659	0.00608486	-0.01248346
2006	1.75097307	5.81178641	0.00581179	-0.01169794
2007	1.82771353	5.54756647	0.04160675	0.023329613
2008	-3.7443058	5.48279252	0.04112094	0.078564002
2009	-2.0357865	5.69217038	0.01707651	0.037434376
2010	3.43607984	5.59231606	0.01677695	-0.01758385
2011	0.29239635	5.67599856	0.017028	0.014104032
2012	0.67644951	6.00550146	0.0180165	0.011252009

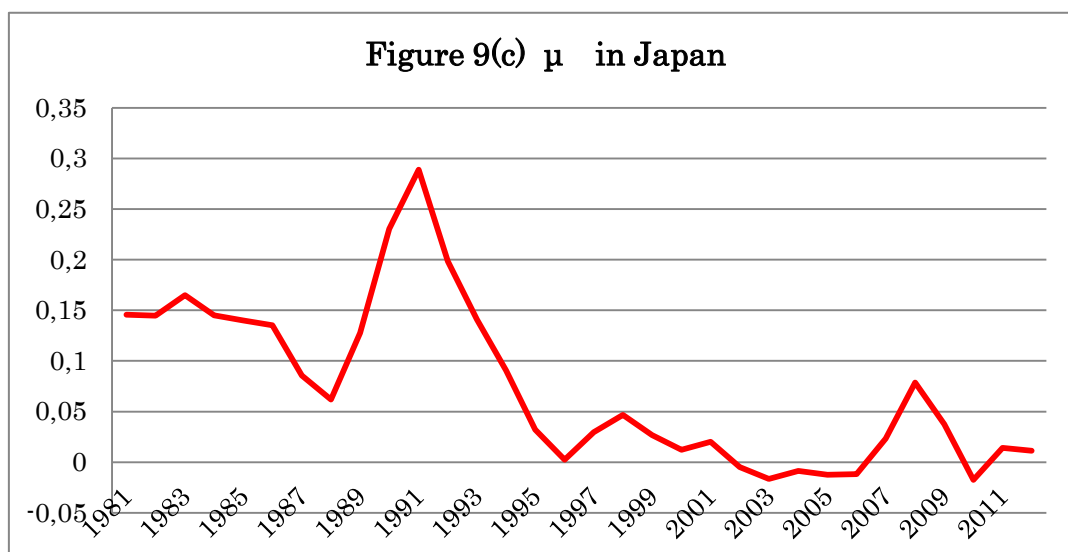
Source; <http://www.boj.or.jp/>

Finally, let us investigate the case of Japan. First, we should indicate that the ratio $a(=M/Y)$ reached a high level, 5.58, in 1989. Japan experienced the collapse of its economic bubble in 1991. However, after 1991, the ratio a continued to increase, only to reach at higher level, 6, in 2005. This trend indicates the Bank of Japan's struggle to

escape from its deep recession. As is well known, this high ratio could not overcome the recession and in fact made matters worse.

As shown in Figure-9(c), the graph of μ has a sharp and high peak in 1991 which corresponds to the collapse of the bubble. The low level of μ during a period of around 20 years corresponds to the so called 'two lost decades of Japan'. During this period, the monetary sector had little power to absorb the value from the real economic sector because the real economic sector has no room to produce enough GDP to be absorbed. More precisely, Japanese firms were initiating activities overseas to pursue low cost production and cultivate new markets.





Finally, let us investigate USA, Euro Area and Japan together. Focusing on μ , the graphs of μ have sharp peaks, which corresponds to the occurrence of financial crashes. This trend is common across all three areas. During the rising segments, the monetary sector absorbed the value from the real economic sector against a background of an expanding monetary base. Following the peaks, the value of μ decreased rapidly. This decrease reflects the rapid decrease of the growth rate, or an economic collapse.

The rapid increases and decreases imply the large deviations from the *natural real interest rate* (*natural distribution rate* of the monetary sector). Therefore, we should conclude that the most important and fundamental causes of financial crisis are the expansion of the monetary base and the deviation from the *natural economy*.

If the economy enters into economic collapse, the government executes fiscal policy by issuing a large amount of national bonds. In addition, the central bank performs substantial amount of buying operations and supplies a large amount of money. This money absorbed the value from the real economic sector. Consequently, the economy was placed on a path to experience more severe collapse. The world economy falls into a *vicious cycle*. The *vicious cycle* in turn deepens the problem.

4.3 Difficulty in Controlling μ

Can we escape from *the vicious cycle*? This depends on the economy's ability to control μ . Let us investigate Equation (29) again. From the viewpoint of financial policy, if a

central bank intends to lower the interest rate, it should increase the monetary base (or the money supply) and vice versa. The increase (decrease) in the monetary base induces an increase (decrease) in the ratio $a(=M/Y)$. However, for instance, if the central bank increases the monetary base, then the interest rate will decrease. That is, in ar in Equation (29), both a and r move in opposite directions when financial policy is implemented. This implies that the central bank faces a contradiction in controlling μ . The difficulty in controlling μ also means the vicious cycle is difficult to escape.

Conclusion

First, we summarize our results. We tried to prove our hypothesis that a fundamental cause of economic crisis is the imbalance between the real economic sector and the monetary sector. To test our hypothesis, we constructed a game-theoretical model and then tried to simulate the distribution rate to the monetary sectors in Euro Area, USA and Japan. This approach yields the following theoretical results.

- (1) The parameter μ , which represents the rate that monetary sector can receive over the growth rate, has a rigorous range for the game to have a solution.
- (2) Our hypothesis is based on the importance of the *natural economy*. That is, it represents a rigorous bench mark around which the real economy can fluctuate. Large deviations from this bench mark gives us a critical alarm. As discussed in the introduction, the *natural economy* involves the concept of the equilibrium in the macro-economy. Taking the contraposition logically, if the economy is not in equilibrium, then it is not a *natural economy*. In addition, as analysed in Section 3, the solution of the game-model has the character of the *natural economy*.
- (3) The simulation of the parameter μ in Section 4 indicates that the rapid expansion of financial assets destroys the balance between the real economic sector and the monetary sector. This fact implies that the current economy loses its balance and deviates largely from the *natural economy*.

Finally, we should indicate some remaining theoretical problems. The first is related to the solution of the game. That is, the model has no stable steady state point. The basic idea for constructing the solution is to confine the auxiliary variables within a finite area. If

players succeed in this task, the optimal and transversality conditions are satisfied simultaneously and the game has a solution. We succeeded in constructing the solution. However, we were unable to propose the conditions for the solution made by *the strategy rule* to converge to the solution that satisfies Equation (23). The solution of Equation (23) has the character of the natural economy. Therefore, the solution yielded by the strategy rule circles around the solution of Equation (23). Conditions with respect to the convergence problem should be studied mathematically.

The second problem is related to the definition of the *natural economy* and *natural interest rates*. As discussed in the introduction, the definition of *natural interest rates* as defined by Wicksell and Keynes are the representative ones. However, the event wherein investment equals savings implies that it depends on the shapes of investment and savings functions. Therefore, if an innovation is expected to occur, the expectation of this innovation will change the investment function. Therefore, we are required to define a *natural economy* able to endure the structural changes caused by innovations. An important clue to this problem lies in the approach made by Pasinetti(1981). That is, the introduction of game theory to the vertically integrated analysis is inevitable to solve this problem. Akimoto(2012) is examining this problem.

The final problem lies in the weakness of the simulation. We used the data of the central banks of each area [See (28)]. M denotes the amount of financial assets held by financial corporations. The parameter a is the ratio of M to GDP. Is it adequate to adopt the amount of financial assets held by financial corporations? In addition, the simulation requires a more precise econometric approach. These remaining problems should be examined to further prove our hypothesis.

References

- Akimoto,K(2012).,“Dynamically Complex Balanced Economic Growth with Innovation ; -A Synthesis of the Theories of Ricardo, Keynes and Schumpeter,” in the proceedings of the conference in 2012-53RSA Matera, the homepage of Italian Economic Association, <http://www.siecon.org/online/en/>(See in Italiano).
- Alesina,A.and Angeletos,G.M. (2005),”Fairness and Redistribution,” *American Economic Review*,Vol.95,960-980.
- Davies,J.B., Sandström,S., Shorrocks,A. and Wolf,E.N.(2008), “The World Distribution of Household Wealth,” *United Nations University Discussion Paper* No. 2008/03
- Basar,J.S. and Olsder, G.J. (1982), *Dynamic Noncooperative Game Theory*, Academic Press.
- Basar,T.,Haurie,A.,and Ricci,G.(1985), “On the Dominance of Capitalists Leadership in a ‘Feedback-Stackelberg’ Solution of a Differential Game Model of Capitalism,” *Journal of Economic Dynamics and Control*, Vol.9. pp.101-125.
- Roland,B. and Tirole,J.(2006),”Belief in a Just World and Redistributive Politics,” *Quarterly Journal of Economics*, Vol.121,pp.699-745.
- Hoel,M.(1978), “Distribution and Growth as a Differential Game between Workers and Capitalists,” *International Economic Review*, Vol.19,No.2,pp.335-350.
- Kaitala,V., and Pohjola,M.(1990),”Economic Development and Agreeable Redistribution in Capitalism: Efficient Game Equilibria in a Two-class Neoclassical Model,” *International Economic Review*,Vol.31,No.2,pp.421-438.
- Keynes, J.M.(1930), *A Treatise on Money 2 ;The Applied Theory of Money*, (The Collected Writings of John Maynard Keynes Vol.VI) The Macmillan Press 1971.
- Keynes, J.M.(1936), *The General Theory of Employment, Interest, and Money* (The Collected Writings of John Maynard Keynes Vol.VII) The Macmillan Press 1971.
- Lancaster,K.(1973), ”The Dynamic Inefficiency of Capitalism,” *Journal of Political Economy*,Vol.81,pp.1092-1109.
- McKinsey & Company (2008) ,“Mapping Global Capital Markets, “ *Fourth Annual Report*, Jan. 2008”.
- Pasinetti,L.L.(1973),”The Notion of Vertical Integration in Economic Analysis,”

Metroeconomica, Vol.25, pp.1-29.

Pasinetti, L.L. (1974), *Growth and Income Distribution, Essays in Economic Theory*, Cambridge University Press.

Pasinetti, L.L. (1977), *Lectures on the Theory of Production*, Columbia University Press.

Pasinetti, L.L. (1981), *Structural Change and Economic Growth; a theoretical essay on the dynamics of the wealth of Nation*, Cambridge University Press.

Pasinetti, L.L. (1990), "Normalized General Coordinates and Vertically Integrated Sectors in a Simple Case," in *Nonlinear and Multisectoral Macrodynamics*, K. Velupillai (ed.), The Macmillan Press.

Pohjola, M. (1983), "Nash and Stackelberg Solutions in a Differential Game Model of Capitalism," *Journal of Economic Dynamics and Control*, Vol.6, pp.173-186.

Pohjola, M. (1985), "Growth Distribution and Employment Modeled as a Differential Games," in *Optimal Control Theory and Economic Analysis 2*, Feichtinger, G. (ed.), North-Holland, 1985.

Pohjola, M. (1986), "Applications of Dynamic Game Theory to Macroeconomics," in *Dynamic Games and Applications in Economics*, Bazar, T. (ed.), Springer-Verlag.

Smith, A. (1776), *An inquiry into the nature and causes of the wealth of nations*, edited, with an introduction, notes, marginal summary and enlarged index by Edwin, M.A., LL.D., professor of political economy in the University of London, 6th edition, 2 vols., London, 1950.

Wicksell, J.F.K. (1898), *Geldzins und Güterpreise, Eine Studie über die den Tauschwert des Geldes bestimmenden Ursachen*, Jena.

<http://www.boj.or.jp/>

<http://www.ecb.europa.eu/home/html/index.en.html>

<http://www.federalreserve.gov/>

<http://www.meti.go.jp/report/tsuhaku2008/.../html/i1120000.html>

<http://www.siecon.org/online/en/>